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SPECTRUM OF AN AFFINE TRANSFORMATION

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The class of measure preserving transformations modelled on topological groups and their homogeneous spaces has been of wide interest in Ergodic theory almost since the beginning of the subject. In the recent years many authors including L. Auslander (cf. [1] and other papers referred there) C. C. Moore [11], W. Parry [12] and also the present author (cf. [3], [4]) studied various ergodic properties of 'translations' and more generally 'affine transformations' of homogeneous spaces of special classes (semisimple, solvable etc.) of groups.

Our object in this article is to study the spectrum of an affine transformation T of G/Γ where G is any Lie group and Γ is a lattice in G; i.e. Γ is a discrete subgroup and G/Γ carries a finite G-invariant measure. For a translation T_g of G/Γ by an element $g \in G$ we find (Theorem 2.3) a closed normal subgroup L of G such that i) T_g has absolutely continuous spectrum on the ortho-complement of the space of L-invariant functions and ii) the factor of the translation determined by L is a 'bundle' of translations of compact abelian groups. This enables us to deduce various criteria for ergodicity, weak mixing, strong mixing and interrelations of these properties. In particular, verification of ergodicity of a translation is reduced to that of two factors which are modelled over homogeneous spaces of a solvable and a semisimple group respectively (Theorem 6.4). Another interesting consequence of our result is that if a homogeneous space admits a weakly mixing translation then every ergodic translation is strongly mixing (Cor. 6.7).

In the last section we extend the study to all affine transformations. (cf. Theorem 7.1). It is deduced that every weakly mixing affine transformation is strongly mixing.

It may be worthwhile to note that under suitable additional conditions (see Remark 5.3) the assertion i) above regarding L may be strengthened to i') T_g has Lebesgue spectrum on the ortho-complement of L-invariant functions, whose multiplicity if non-zero is infinite.

§1. Preliminaries.

A Lie group G, by convention may have finitely many connected components. A connected Lie group (respectively a connected Lie subgroup) is also referred as an analytic group (analytic subgroup). The connected component of the identity in a Lie subgroup H is denoted by H°.

A *lattice* Γ in a Lie group G is a discrete subgroup of G such that G/Γ admits

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