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MEASURABLE NORMS AND SOME BANACH SPACE VALUED GAUSSIAN PROCESSES

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1. Introduction.

In recent years, and especially since E. Nelson introduced and studied the free Markov field, the study of probability measures on topological vector spaces has taken on great importance in quantum field theory. The free field measure has already been extensively studied (see for example [23], [24], [6] and [2]); nevertheless it is the main subject of the investigations of this paper. Although we will also obtain support properties, our point of view is different from the one adopted in [23], [24] and [6]: instead of looking at the free field measure as the distribution of a real valued Gaussian process indexed by the test function space $\mathcal{S}(\mathbf{R}^{d+1})$, we view it as the distribution of a Gaussian process indexed by $\mathbf{R}$ and taking its values in an infinite dimensional space—a process which can be considered as a Banach space valued Ornstein–Uhlenbeck process ([2] and [21]).

This point of view was proved to be useful in the study of the Yukawa model, especially since it is possible to prove the continuity of the paths (see [16]).

We will use the concepts of measurable norm and of abstract Wiener space due to L. Gross. A measurable norm on a real separable Hilbert space is defined [13] in terms of the canonical measure of particular cylinder sets and the fundamental result [14] asserts that the canonical measure becomes σ-additive on the completion of the Hilbert space with respect to the given measurable norm. Conversely, [9], given any real separable Banach space B and any Gaussian probability measure μ the support of which is B, it is possible to build a dense set in B which can be equipped with a Hilbert structure in such a way that the B-norm is measurable on H, and μ appears as the extension of the canonical cylindrical measure. These facts are reviewed in section 2: all the results of this section are known or essentially known, the main novelty being the short proof we give of the above mentioned converse.

Now, let A be the pseudo-differential operator $(-\Delta + 1)^{1/2}$ on the Sobolev space $\mathcal{H}^d_{-1/2}$ and let B be the completion of the dual space $\mathcal{H}^d_{1/2}$ with respect to a measurable norm. We may expect the free field measure to be the distribution of a B-valued Gaussian process $\{X_t\ ; t \in \mathbf{R}\}$ which satisfies:

$$E\{\langle f, X_t\rangle\langle g, X_s\rangle\} = (e^{-|t-s|A}f, g)^d_{\mathcal{H}_{-1/2}}$$

for any real numbers s and t and for any f and g in the dual of B.

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