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THE BOUNDARY VALUES OF HOLOMORPHIC FUNCTIONS OF SEVERAL COMPLEX VARIABLES

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It is a result of Agranovskii and Val'skii [1] for which Nagel and Rudin [5] have recently given an alternative proof that if $f \in \mathcal{C}(\partial B_n)$, B_n the open unit ball in $\mathbf{C}^n$, has the property that for every complex line $\Lambda \subset \mathbf{C}^n$, $f \mid (\partial B_n \cap \Lambda)$ extends holomorphically into the disc $\Lambda \cap B_n$, then f extends holomorphically into B_n. The proofs for this result, though quite different from each other, have in common the property of depending on the symmetry of B_n, on the fact that B_n has a large group of holomorphic automorphisms. Thus, they do not extend in an obvious way to more general domains. Rudin has proposed the possibility of deriving the result from Weinstock's characterization [8] of the boundary values of holomorphic functions. In this note, we shall show that such a derivation is possible and that the result extends to general smoothly bounded domains. The main analytic tool in our discussion is the Radon transform in the complex domain as developed in [2]. (Cf. [3].)

Let $D \subset \mathbf{C}^n$ be a bounded domain with boundary of class C^2. We shall say that an $f \in \mathcal{C}(\partial D)$ has the *one-dimensional extension property* if for every complex line Λ in $\mathbf{C}^n$, there is $F_\Lambda \in A(\Lambda \cap D) = \{F \in \mathcal{C}(\Lambda \cap \bar{D})$: F is holomorphic on the interior of $\Lambda \cap \bar{D}\}$ with $F_\Lambda = f$ on $\Lambda \cap \partial D$. (Of course, *interior* here is taken with respect to the line Λ, not with respect to the ambient $\mathbf{C}^n$.)

THEOREM. *If $D \subset \mathbf{C}^n$ is a bounded domain with $\mathcal{C}^2$ boundary, then every $f \in \mathcal{C}(\partial D)$ with the one-dimensional extension property extends continuously to $\bar{D}$ so as to be holomorphic in D.*

Proof. We shall show that $\int_{\partial D} f \bar{\partial} \alpha = 0$ for all $\mathcal{C}^\infty$ forms α of bidegree $(n, n-2)$ defined on a neighborhood of $\bar{D}$. If ∂D is connected, this yields, *via* a theorem of Weinstock [8], the desired extension of f.

In the case of general domains, the desired extension is still possible as shown by the following argument which was suggested by Reese Harvey. The condition $\int_{\partial D} f \bar{\partial} \alpha = 0$ for all compactly supported $\mathcal{C}^\infty$ forms of bidegree $(n, n-2)$ is equivalent to the condition that the current $f[\partial D]^{0,1}$ be $\bar{\partial}$-closed which, according to [4, Lemma 3.11] implies the existence of a unique compactly supported distribution F with $\bar{\partial} F = f[\partial D]^{0,1}$ which is, in fact, an L_1-function. By [4, Theorem 3.18], the function f will admit the desired extension provided F is supported in $\bar{D}$. That supp $F \subset \bar{D}$ follows from the one-dimensional

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