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# EXTENSION OF PERIOD MAPPINGS FOR HODGE STRUCTURES OF WEIGHT TWO

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**Introduction.**

In his survey on periods of integrals on algebraic manifolds ([6]), Griffiths conjectured the existence of a partial compactification for the arithmetic quotient $\Gamma\backslash D$ of a classifying space for polarized Hodge structures, with the property that the holomorphic mappings arising from variations of Hodge structures (Griffiths' period mapping) extend continuously across the singularities of the parameter space. In the case of Hodge structures of weight one, $D$ is a Siegel upper half space and, consequently, a solution is given by the Satake-Baily-Borel ([1], [9]) compactification, together with Borel's extension theorem ([3]).

In this paper we answer the conjecture in the affirmative for the case of Hodge structures of weight two, where $D = O(2h, k)/U(h) \times O(k)$, and period mappings of one complex parameter. The restriction to weight two is in some sense natural since these classifying spaces reflect the essential features appearing in the arbitrary weight case, while at the same time allow for relatively simple explicit constructions. In fact, the same ideas, together with a good part of the statements and proofs in this paper, should carry through to the general case. This will be treated in a future paper.

We construct a locally compact, Hausdorff space $\Gamma\backslash D^{**}$, containing $\Gamma\backslash D$ as an open dense subset such that every horizontal, locally liftable map from the punctured unit disc into $\Gamma\backslash D$ extends continuously, over the puncture, to a map into the partial compactification $\Gamma\backslash D^{**}$; this space is also minimal relative to this property. The extended space $D^{**}$ is the union of $D$ and the rational "boundary components"; these are products of classifying spaces of Hodge structures of weight one and two, and their definition is motivated by the results of Schmid on the singularities of the period mapping. In [10] it is shown that when a one-parameter family of Hodge structures degenerates, the Hodge filtration "at infinity" together with the monodromy weight filtration defines a mixed Hodge structure. The boundary components defined here contain the information given by the polarized Hodge structures that the filtration at infinity defines in the primitive part of the graded quotients of the weight filtration.

In order to define a Satake topology on $D^{**}$ it is useful to introduce the notion of boundary bundles (loosely speaking these are fibre bundles having the

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