Vol. 43, No. 4 DUKE MATHEMATICAL JOURNAL© December 1976

A NOTE ON A RESULT OF J. BRAM

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In [1] J. Bram proved the following:

THEOREM 1. *Let A be a subnormal operator on a Hilbert space H and B its minimal normal extension on $K \supseteq H$. Then necessary and sufficient conditions that a bounded operator T_0 on H have an extension T on K such that T commutes with B are:*

(*a*) *T_0 commutes with A.*

(b) *For every finite set $\{x_i\}_{i=0}^n$, $x_i \in H$,*

$$\sum_{m,n=0}^{r} \langle A^m T_0 x_n , A^n T_0 x_m \rangle \leq c \sum_{m,n=0}^{r} \langle A^m x_n , A^n x_m \rangle$$

where c is fixed.

If the extension T exists, it is unique.

Condition (*a*) is implied by (*b*) and is therefore superfluous. To see this we modify Bram's proof as follows: Let $\mathfrak{D}$ be the set of vectors in K of the form $\sum_{n=0}^{r} B^{*n} x_n$ for some r and $x_i \in H$ and define

$$T\left(\sum_{n=0}^{r} B^{*n} x_n\right) = \sum_{n=0}^{r} B^{*n} T_0 x_n$$

Bram's proof that T is well-defined and bounded only uses (b). Now notice that

$$TB^*\left(\sum_{n=0}^{r} B^{*n} x_n\right) = T\left(\sum_{n=0}^{r} B^{*n+1} x_n\right) = \sum_{n=0}^{r} B^{*n+1} T_0 x_n$$

$$= B^*\left(\sum_{n=0}^{r} B^{*n} T_0 x_n\right) = B^*T\left(\sum_{n=0}^{r} B^{*n} x_n\right)$$

so $[T, B^*] = 0$. Since B is normal we have $[T, B] = 0$ which implies $[T_0 , A] = 0$ on H.

This remark also applies to Itô's generalization of Bram's result [2].

REFERENCES

1. J. BRAM, *Subnormal operators,* Duke Math. J. 22(1955), 75–94

2. T. ITÔ, *On the commutative family of subnormal operators,* J. Fac. Sci. Hokkaidô Univ. 14(1958), 1–15.

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Received September 15, 1976.