Vol. 43, No. 4 DUKE MATHEMATICAL JOURNAL© December 1976

ON KÄHLER MANIFOLDS WITH NEGATIVE HOLOMORPHIC BISECTIONAL CURVATURE

PAUL C. YANG

§0. Introduction.

In [2], Greene and Wu proved that if M is a complete Kähler manifold of non-positive riemannian curvature, then its universal cover $\tilde{M}$ is a Stein manifold. It is generally suspected that $\tilde{M}$ should be a bounded domain in C^n if additional restrictions are imposed on the curvature of M (see [2], [3]). It is our purpose in this note to observe some restrictions on $\tilde{M}$ under certain curvature assumptions on M. In particular, we apply variational formula of volume integral to prove the following:

THEOREM. *The polydisc* D^n, $(n > 1)$, *does not admit a complete Kähler metric with its holomorphic bisectional curvature bounded between two negative constants* $-c \leq K(\sigma, \sigma') \leq -d < 0$.

The following is an immediate consequence:

COROLLARY. *If* M^n $(n > 1)$ *is a compact Kähler manifold with negative bisectional curvature, then its universal cover* $\tilde{M}$ *cannot be a polydisc.*

Remark. It should be clear that the proof of the theorem works for bounded symmetric domain of rank $n > 1$ instead of a polydisc, hence the corollary remains valid when "polydisc" is replaced by such domains.

Preliminary and notations. We shall follow the notations of [4]. For definition of holomorphic bisectional curvature and its properties see [1].

We shall need the following version of the Schwarz lemma due to Yau [5].

Schwarz lemma. Let M be a complete Kähler manifold with Ricci curvature bounded from below by a constant and N be another Kähler manifold with holomorphic bisectional curvature bounded above by a negative constant. Then any holomorphic mapping from M into N decreases distances up to a constant depending on the bounds of curvatures of M and N.

§1. A variation formula.

Let $(z_1, \cdots, z_n) = (z, w)$, $z = (z_1, \cdots, z_{n-1})$, $w = z_n$ be the coordinates on $D^n = D^{n-1} \times D$, the unit polydisc. Suppose

$$ds^2 = \sum_{1 \leq i, j \leq n} g_{i\bar{j}}\, dz_i\, d\bar{z}_j$$

Received September 20, 1976. Research supported by NSF Grant MPS 75-05270.