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ON UNIFORMLY REGULAR TOPOLOGICAL MEASURE SPACES

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I. Introduction.

Let $(X, \mathfrak{F}, \mu)$ and $(X', \mathfrak{F}', \mu')$ be two abstract measure spaces and suppose that there is a measurable map $p : X \to X'$ such that $\mu' = p(\mu)$, $p(E) \in \mathfrak{F}'$ for all $E \in \mathfrak{F}$, and $\mu'[p(E)] = \mu(E)$. Then clearly the associated measure algebras are isomorphic, so that measure theoretic properties of $(X', \mathfrak{F}', \mu')$ can be lifted to $(X, \mathfrak{F}, \mu)$. In some sense $(X', \mathfrak{F}', \mu')$ represents $(X, \mathfrak{F}, \mu)$. In this note, the concept of representation is developed in a topological set up and is tnen used to:

(a) characterize uniformly regular measures on compact spaces as introduced in [1];

(b) provide a stock of examples of non-metrizable compact spaces supporting uniformly regular measures;

(c) give a topological version of a result due to Zink [11], which is itself a refinement of a theorem of Halmos and Von-Neumann.

The property of uniform regularity of a measure on a compact space X which is naturally possessed by all Borel measures on metric spaces, ties in the measure more closely with the topological and uniform structure of X. Roughly speaking, it makes the measure look as though it is living in a compact metric space.

Throughout, unless otherwise explicitly stated, X will denote a compact Hausdorff space. All unexplained notions concerning the topology and the uniform structure of X are those of [5] and [7]; the measure theoretic ones are those of [3] and [6].

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II. Preliminary material.

Let X be a completely regular Hausdorff space. The Baire (Borel) sets of X are elements of the minimal σ-algebra of subsets of X containing all the zero (closed) sets. By a Baire (Borel) measure on X we mean a totally finite, non-negative, countably additive set function defined on the Baire (Borel) sets. Furthermore, we assume that measures are regular in the sense of inner approximation by zero sets in the Baire case and closed sets in the Borel case.

A Baire measure μ is said to be *τ-additive* if for any net $\{Z_\alpha\}_{\alpha \in A}$ of zero subsets of X with $Z_\alpha \searrow \varnothing$, we have $\mu(Z_\alpha) \to 0$. μ is called *tight* if for any $\epsilon > 0$ there is

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