Vol. 43, No. 3 DUKE MATHEMATICAL JOURNAL© September 1976

THE COHOMOLOGY OF CERTAIN SPECTRA ASSOCIATED WITH THE BROWN-PETERSON SPECTRUM

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0. Introduction.

For a fixed prime p, W. Stephen Wilson has defined a sequence of extraordinary cohomology theories $BP\langle n\rangle^*(\ )$, $n = 0, 1, 2, \cdots \infty$ [12]. $BP\langle 0\rangle^*(X) = H^*(X; Z_{(p)})$. $BP\langle 1\rangle^*(X)$ is a summand of the complex connective K-theory of X localized at p. $BP\langle\infty\rangle^*(X) = BP^*(X)$ is the Brown-Peterson cohomology of X. Let $BP\langle n\rangle = \{BP\langle n\rangle_k\}$ be an omega spectrum representing the cohomology theory $BP\langle n\rangle^*(\ )$: for a finite complex X, $BP\langle n\rangle^k(X) \cong [X, BP\langle n\rangle_k]$. We adopt the convention that $H^*X = H^*(X; Z_p)$ (Z_p denotes the integers mod p). Let $\mathcal{A}$ be the mod p Steenrod algebra and Q_i the Milnor elements [3]. W. Stephen Wilson [12] proved that $H^*BP\langle n\rangle \cong \mathcal{A}/\mathcal{A}(Q_0, Q_1, \cdots Q_n)$. The object of the paper is to compute the mod p cohomology of the spaces $BP\langle n\rangle_k$. Our result is:

THEOREM.

$$H^*BP\langle n\rangle_{2k} = Z_p[vb(I, J)] \otimes F[\overline{M^n{}_{s(n,k-1)}}]$$

and

$$H^*BP\langle n\rangle_{2k+1} = \Lambda[\sigma vb(I, J)] \otimes F[M^n{}_{s-1}] \otimes \Lambda\left[\frac{(M^n{}_s)^+ \cap \phi(M^{n-1}{}_s)^-}{\phi(M^n{}_s)^+}\right]$$

The algebra generators $vb(I, J)$ satisfy the technical requirement that $vb(I, J)$ be (n, k)-allowable. $F[M]$ is the free commutative algebra on the graded vector space M. We defer the technical description of the modules $M_s{}^n$ and of the polynomial and exterior algebra generators until Section *2*.

Our proof is by double induction on (n, k) which exploits the antecedents of this theorem. Cartan's computation of $H^*K(Z, k)$ [1] provides the $(0, k)$ case of the theorem and begins the induction in the first coordinate. Ravenel and Wilson [6] have computed the homology of the Brown-Peterson omega spectrum spaces $BP\langle\infty\rangle_k$. By Wilson's Splitting theorem [12], this yields a calculation of $H^*BP\langle n+1\rangle_{2k}$ for low k. This gives a starting point for the induction in k for a fixed $n + 1$. Two other antecedents—the $n = 1$ case of the theorem—remain. Stong [10] computed the mod 2 cohomology of the spectra associated with BO and BU. Stong's results served as a guide to William Singer [8] who computed the mod p cohomology of the connective coverings of BU and

Received December 10, 1975. Revision received May 29, 1976.