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# REMARKS ON CURVATURE ESTIMATES FOR MINIMAL HYPERSURFACES

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In [6] curvature estimates (both pointwise and integral estimates) were obtained for stable minimal hypersurfaces immersed in a Riemannian manifold. In particular, a pointwise estimate was established for boundaries of least area contained in $\mathbf{R}^{n+1}$ for $n \leq 5$.

Here (Theorem 1) we wish to point out that such a pointwise estimate can be obtained by a very simple argument, based on standard regularity theory for minimal hypersurfaces, in case $n \leq 6$. The result also holds for $n = 7$ in the non-parametric case, when the hypersurface is the graph of some solution $u$ of the minimal surface equation. That the result cannot hold for $n > 7$ follows from [3] (See remark 3 below).

We also here show (Theorem 2) that for a given non-parametric minimal hypersurface $x_{n+1} = u(x_1, \cdots, x_n)$, with $n \geq 2$ arbitrary, there is an interesting necessary and sufficient condition for the existence of a curvature bound, involving a Harnack inequality for $(1 + u_x^2)^{\frac{1}{2}}$ ($u_x = (u_{x_1}, \cdots, u_{x_n})$ = gradient $u$).

By combining Theorems 1 and 2 we thus establish a Harnack inequality for $(1 + u_x^2)^{\frac{1}{2}}$ for any solution $u$ of the minimal surface equation in the case $n \leq 7$. (See Theorem 3.)

We use the following notation:

$$B_\rho = \{x \in \mathbf{R}^{n+1} : |x| < \rho\}$$

$$D_\rho = \{x \in \mathbf{R}^n : |x| < \rho\}$$

$$\omega_n = \text{volume of } D_1$$

$H_n$ denotes $n$-dimension Hausdorff measure in $\mathbf{R}^{n+1}$

$\mathcal{O}$ will denote the collection of all open sets

$U \subset \mathbf{R}^{n+1}$ having the property

$$O \in \partial U \cap B_1 = \partial \bar{U} \cap B_1.$$

Notice that if $U \in \mathcal{O}$ and $\partial U \cap B_1$ is smooth, then (in $B_1$) it makes sense to speak of the outward unit normal to $U$, so that all smooth elements of $\mathcal{O}$ have naturally oriented boundaries in $B_1$.

$\mathcal{U}$ will denote the collection of $U \in \mathcal{O}$ such that $\partial U \cap B_1$ is $C^2$ and such that $\partial U$ has least area in $B_1$ in the sense that

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