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AUTOMORPHISMS OF FULL II_1 FACTORS, WITH APPLICATIONS TO FACTORS OF TYPE III

JOHN PHILLIPS

Introduction.

In this paper we study outer conjugacy classes of automorphisms of a certain von Neumann factor of type $\mathbf{II}_1$. This study is motivated by A. Connes' classification of type $\mathbf{III}_\lambda$ factors, λ in $]0, 1[$. In this scheme, he reduces the problem to one of classifying, up to outer conjugacy, certain automorphisms of factors of type $\mathbf{II}_\infty$.

In particular, we exhibit a Borel family of automorphisms of a certain $\mathbf{II}_1$ factor for which the outer conjugacy classes are not countably separated. Using this result and the classification of $\mathbf{III}_\lambda$ factors, we show that for each λ in $]0, 1[$, the isomorphism classes of $\mathbf{III}_\lambda$ factors are not countably separated.

We also investigate other automorphisms of this full $\mathbf{II}_1$ factor, which are not outer conjugate to any of the previously defined automorphisms.

Preliminaries.

If A is a von Neumann algebra, we denote the group of *-automorphisms of A by Aut A and its normal subgroup of inner automorphisms by Int A. We let ϵ denote the canonical homomorphism, ϵ: Aut $A \to$ Aut A/Int A and we denote the quotient group Aut A/Int A by Out A.

We let A_* denote the predual of A and endow Aut A with the topology of pointwise norm convergence in A_*. With this topology, Aut A is a topological group which is polish if A_* is separable, [10]. A is called full if Int A is closed in this topology. If A is a $\mathbf{II}_1$ factor with canonical trace, tr, then the topology on Aut A is actually the topology of pointwise convergence in A, where A is given the norm $||x||_2^2 = \text{tr}(x^*x)$, [10].

Let F_n be the free nonabelian group on n generators ($n = 2, 3, \cdots, +\infty$). Then, F_n is a countable discrete group with infinite conjugacy classes. For each g in F_n, let $\lambda(g)$ be the unitary operator on $\ell^2(F_n)$ defined by:

$$(\lambda(g)f)(h) = f(g^{-1}h), \qquad f \text{ in } \ell^2(F_n), \quad h \text{ in } F_n .$$

Let $\mathfrak{U}(F_n) = \{\lambda(g) \mid g \text{ in } F_n\}''$ be the left von Neumann algebra of F_n on $\ell^2(F_n)$. It is well known that $\mathfrak{U}(F_n)$ is a factor of type $\mathbf{II}_1$ (this only depends on the fact that F_n has infinite conjugacy classes). Using theorem 2.2 of [12], one can show that any automorphism of $\mathfrak{U}(F_n)$ which is obtained from a nontrivial permutation of the generators of F_n, must be outer. From this, and

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