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A GENERALIZATION OF DIRICHLET'S CLASS NUMBER FORMULA

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CONTENTS

1. Introduction.

In [3], we studied various generalizations of the function $\log \eta(z)$, where $\eta(z)$ is Dedekind's η-function. We called these functions *Hecke integrals*. Among our examples of Hecke integrals was the function $f_\chi(z)$ which is defined as follows: Let χ be an odd, primitive, Dirichlet character of conductor $\mathfrak{b}$, and let $\mathbf{H}$ denote the complex upper half-plane. Then we set

$$f_\chi(z) = \sum_{n=1}^{\infty} \left(\chi(n) \sum_{d \mid n} \frac{\bar{\chi}^2(d)}{d}\right) e^{2\pi i n z/\mathfrak{b}},$$

where $\bar{\chi}$ denotes the complex conjugate character of χ. We showed in [3] that $f_\chi(z)$ satisfies the functional equations

$$f_\chi(z + \mathfrak{b}) = f_\chi(z) \tag{1.1}$$

$$f_\chi\left(-\frac{1}{z}\right) = -f_\chi(z) + \frac{i\mathfrak{b}}{\pi\tau(\bar{\chi})} L(1, \bar{\chi})^2 \tag{1.2}$$

where $L(s, \chi)$ is the usual Dirichlet L-series and $\tau(\chi)$ denotes the Gaussian sum attached to χ.

We discussed in [3] the relationship between the problem of estimating $f_\chi(i)$ and that of determining all imaginary quadratic fields having a given class number. In this paper, we discuss another connection of the function $f_\chi(z)$ with the arithmetic of imaginary quadratic fields.

It is clear from (1.1) and (1.2) that if $\sigma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ belongs to the subgroup $G(\mathfrak{b})$ of $SL(2, \mathbf{R})$ generated by

$$\pm\begin{pmatrix} 1 & \mathfrak{b} \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad \pm\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},$$

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