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SMOOTH ZERO SETS AND INTERPOLATION SETS FOR SOME ALGEBRAS OF HOLOMORPHIC FUNCTIONS ON STRICTLY PSEUDOCONVEX DOMAINS

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§1. Introduction.

Let $\Omega \subset\subset \mathbf{C}^m$ be a strictly pseudoconvex domain with C^3 boundary $\partial\Omega$. For $k = 0, 1, \cdots, \infty$, let $A^k(\bar{\Omega})$ denote the algebra of functions of class C^k on $\bar{\Omega}$ which are holomorphic on Ω. Let $K \subset \partial\Omega$ be a compact set. Then we shall say that K is a *zero set* for algebra $A^k(\bar{\Omega})$ if there exists $F \in A^k(\bar{\Omega})$ with $F(z) \neq 0$ for all $z \in \bar{\Omega}\backslash K$, and $D^\alpha F(z) = 0$ for $z \in K$ and all derivatives D^α of order $|\alpha| \leq k$. We shall say that K is an *interpolation set* for the algebra $A^k(\bar{\Omega})$ if the restriction map $A^k(\bar{\Omega}) \rightarrow C^k(K)$ is onto, where $C^k(K)$ denotes the space of Whitney C^k-functions on K, viewed as a subset of the manifold $\partial\Omega$.

In general, no necessary and sufficient conditions seem to be known for a set K to be a zero set or an interpolating set. However, in the case that $\Omega \subset \mathbf{C}$ is the open unit disc, a complete characterization is known. For $k = 0$, the characterization of zero sets is a theorem of Fatou [6], and the characterization of interpolation sets is due to Carleson [2] and Rudin [10]. For $0 < k < \infty$, the characterization of zero sets is due to Carleson [3], and for $k = \infty$, it is due to Novinger [9] and Taylor and Williams [12]. The characterization of interpolation sets for $A^\infty(\Omega)$ is due to Alexander, Taylor and Williams [1].

For domains in $\mathbf{C}^m$, $m > 1$, much less is known, but some sufficient conditions have been given by Davie and Øksendal [5] for interpolation sets for $A^0(\bar{\Omega})$, and by Chollet [4] for zero sets of $A^\infty(\bar{\Omega})$. In both cases, the condition is a metric condition on the set K.

The main object of this paper is to study smooth subsets of $\partial\Omega$, or more precisely, compact subsets K of not necessarily closed real submanifolds $M \subset \partial\Omega$. Since M is a submanifold, we can impose a directional condition on it at each point: if $T_\zeta\, \partial\Omega$ denotes the real tangent space to $\partial\Omega$ at ζ, if $P_\zeta = (T_\zeta\, \partial\Omega) \cap i(T_\zeta\, \partial\Omega)$ denotes the maximal complex subspace of $T_\zeta\, \partial\Omega$, and if $T_\zeta M$ denotes the real tangent space to M at ζ, we say that M points in the complex direction at ζ if $T_\zeta M \subset P_\zeta$. Two of our main results are then:

A) If $M \subset \partial\Omega$ points in the complex direction at every point, then every compact subset of M is an interpolation set for $A^0(\bar{\Omega})$.

B) If $M \subset \partial\Omega$ points in the complex direction at every point, if $K \subset W \subset M$

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