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SEPARABLE NUCLEAR C^*-ALGEBRAS AND INJECTIVITY

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1. Introduction.

We recall that a C^*-algebra A is said to be *nuclear* if for all C^*-algebras B, the natural *-homomorphism

$$A \otimes_{\max} B \to A \otimes_{\min} B$$

is a *-isomorphism. The authors proved in [2, Th. 3.1] that A is nuclear if and only if there is a net of finite rank completely positive contractions $\varphi_\nu : A \to A$ which converge to the identity mapping in the point-norm topology (the sufficiency of this condition is due to Lance [12, Th. 3.6].) On the other hand, if R is a von Neumann algebra, R is said to be *semidiscrete* if there is a net of finite rank, completely positive, σ-weakly continuous contractions $\psi_\nu : R \to R$ which converge to the identity map in the point-σ-weak topology (see [10, §3] and Lemma 2.1 below). This property also has several tensor product formulations (see [10, §4] and Lemma 2.1).

A natural problem that was first considered in [17, §3] and [10, §6] is the following

CONJECTURE. *If A is a C^*-algebra, and A^{**} is its enveloping von Neumann algebra, then the following are equivalent:*

(a) *A^{**} is semidiscrete,*

(b) *A is nuclear,*

(c) *A^{**} is injective.*

It was shown in [10, Th. 6.4] that (a) $\Rightarrow$ (b) $\Rightarrow$ (c) (for the non-unital case one may use [2, Th. 3.1] and Lemma 2.1 below). The conjecture is of considerable interest since A^{**} will be semidiscrete (resp., injective) if and only if the weak closure of A in any representation has that property. In addition, Tomiyama has pointed out that an affirmative answer would imply that quotients and extensions of nuclear C^*-algebras are again nuclear (see [16, Prop. 8.4] and Corollary 3.3).

Given a nuclear C^*-algebra A and corresponding maps $\varphi_\nu : A \to A$, one might be tempted to argue that the maps $\psi_\nu = \varphi_\nu^{**} : A^{**} \to A^{**}$ could be used to prove that A^{**} is semidiscrete. However, there seems to be no reason to believe that the maps ψ_ν would converge on elements not in A.

Very recently, the authors received a preprint of [3] from A. Connes. In this truly remarkable paper, Connes has shown among other things that if R is a

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