

THE RIEMANN-LEBESGUE PROPERTY FOR ARBITRARY
 LOCALLY COMPACT GROUPS

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In their paper on the Riemann-Lebesgue theorem for abelian groups, cf., [8], Goldberg and Simon deduce the aforementioned theorem from the following *R-L* property (which they show is true for any locally compact abelian group G).

R-L PROPERTY. Let G be a locally compact abelian group with dual group \hat{G} . It is said that G has the *R-L property* if for any neighborhood U of the identity e of G there exists a compact set K in \hat{G} such that, if $\gamma \in \hat{G} - K$, then there exists $x_\gamma \in U$ with $\text{Re } \gamma(x_\gamma) \leq 0$. We say K is a *compact set corresponding to U* .

We begin our paper by giving an improved statement of the *R-L* property for abelian groups which we show is best possible in the general case. We then proceed to establish an *R-L* property for an arbitrary locally compact group G , as well as a dual *R-L* property which is valid for amenable G . We then apply these *R-L* properties to characterize complex Borel measures whose Fourier-Stieltjes transforms vanish at infinity (for general G) as well as characterize those $b \in B(G)$, the Fourier-Stieltjes algebra of G , which vanish at infinity (for amenable G). We also give a new characterization of $A(G)$ the Fourier algebra, for amenable G .

1. The R-L property in the abelian case.

The above mentioned *R-L* property can be refined thusly.

Definition 1. Let G be a locally compact abelian group with dual group \hat{G} . It is said that G has the (*refined*) *R-L property* if for any neighborhood U of the identity e of G there exists a compact set K in \hat{G} such that, if $\gamma \in \hat{G} - K$, then there exists $x_\gamma \in U$ with $\text{Re } \gamma(x_\gamma) \leq -1/2$. (Note that K is called a *compact set corresponding to U* .)

Remark. Intuitively what this property states is that one obtains an arbitrarily high “frequency” of oscillation for all $\gamma \in \hat{G}$ sufficiently “far” from the identity of \hat{G} .

PROPOSITION 1. *Every locally compact abelian group has the (refined) R-L property.*

Proof. This result may be obtained by making the appropriate minor changes

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