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MACKEY TOPOLOGIES, REPRODUCING KERNELS, AND DIAGONAL MAPS ON THE HARDY AND BERGMAN SPACES

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1. Introduction. Let U denote the open unit disc in the complex plane, T the unit circle; and $d\lambda$ and dm Lebesque measure on U and T respectively, both normalized to have total mass 1. The *Hardy space* H^p consists of all functions f analytic in U for which

$$||f||_p^p = \sup_{0 \le r < 1} \int_T |f(r\omega)|^p \, dm(\omega) < \infty,$$

and the *weighted Bergman space* A_α^p ($\alpha > -1$) consists of those f analytic on U for which

$$||f||_{p,\alpha}^p = \int_U |f(z)|^p (1 - |z|)^\alpha \, d\lambda(z) < \infty.$$

If $1 \le p < \infty$ these are Banach spaces in the obvious norms. In this paper, however, we are interested in the range $0 < p < 1$, in which case the appropriate metrics are

$$d(f, g) = ||f - g||_p^p \quad \text{for} \quad H^p$$

and

$$d(f, g) = ||f - g||_{p,\alpha}^p \quad \text{for} \quad A_\alpha^p.$$

These metrics turn H^p and A_α^p respectively into *F-spaces* (complete, metrizable linear topological spaces) which are *not* locally convex, but nevertheless have enough continuous linear functionals to separate points (the evaluation functionals $f \to f(z)$ for $z \in U$, for example: see [1, page 37 and sec. 7.4] and [8] for H^p, [11] for A_α^p, and also section 2 of this paper).

In 1932 Hardy and Littlewood showed that $H^p \subset A_{1/p-2}^1$ for $0 < p < 1$, the inclusion map being continuous [5; Theorem 31, pp. 411–412] (reassurance: in this paper $a/b - c$ always means $(a/b) - c$). Recently Duren, Romberg, and Shields [2; Theorems 1 and 7] explicitly determined the dual spaces of H^p and $A_{1/p-2}^1$; and found them to be *the same* in the sense that every continuous linear functional on $A_{1/p-2}^1$ restricts to one on H^p, and every continuous linear functional on H^p extends uniquely to one on $A_{1/p-2}^1$. In short, the restriction

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