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ON THE GROWTH OF MEROMORPHIC FUNCTIONS HAVING AT LEAST ONE DEFICIENT VALUE

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Introduction. Let $f(z)$ be meromorphic in the plane and denote by $N(r, c)$ the usual Nevanlinna counting function for the c-points of f in $|z| \leq r$ (for this and other standard terminology, see [10]). If f has finite order λ and

$$m_2(r, f) = \left\{\frac{1}{2\pi} \int_0^{2\pi} (\log |f(re^{i\theta})|)^2 \, d\theta\right\}^{\frac{1}{2}},$$

then we have shown [14]

$$\limsup_{r\to\infty} \frac{N(r, 0) + N(r, \infty)}{m_2(r, f)} \geq C(\lambda) \qquad (0 \leq \lambda < \infty), \tag{1}$$

$$C(\lambda) = \frac{|\sin \pi\lambda|}{\pi\lambda} \left\{\frac{2}{1 + (\sin 2\pi\lambda)/2\pi\lambda}\right\}^{\frac{1}{2}}. \tag{2}$$

Equality holds in (1) for "Lindelöf functions": entire f having all zeros on a ray through 0 and $N(r, 0) \sim r^{\lambda}$ $(r \to \infty)$, see [15, p. 229].

If we take into account the lower order

$$\mu = \liminf_{r\to\infty} \frac{\log T(r, f)}{\log r}$$

of f, where T denotes the usual Nevanlinna characteristic, results of Edrei [3], Kjellberg [13] and Ostrovskii [16] suggest that (1) can be extended in a useful (e.g., [9, pp. 121, 123]) way, to

$$\limsup_{r\to\infty} \frac{N(r, 0) + N(r, \infty)}{m_2(r, f)} \geq C(\rho) \qquad (\mu \leq \rho \leq \lambda) \tag{3}$$

if $\mu < \infty$, where C is defined in (2).

However, the methods of [3], [13], [16] are not applicable here because of an interesting technical difficulty (see Section 1); our solution uses a method that makes Edrei's notion of Pólya peaks more flexible in other problems as well.

Let $\{r_n\}$ be a sequence of *Pólya peaks of order* ρ *for* $T(r) = T(r, f)$, i.e., $r_n \to \infty$ and

$$T(r) \leq T(r_n)(r/r_n)^{\rho}(1 + \eta_n) \qquad (\eta_n r_n \leq r \leq \eta_n^{-1} r_n) \tag{4}$$

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