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COEFFICIENT OF INJECTIVITY FOR QUASIREGULAR MAPPINGS

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1. Introduction.

1.1. Let G be a domain in the n-dimensional euclidean space R^n, $n \geq 2$. Consider a local homeomorphism $f : G \to R^n$. For every $x \in G$ we define $r(x, f)$ as the supremum over all $r > 0$ such that f is injective in

$$B^n(x, r) = \{y \in R^n \mid |x - y| < r\}$$

and $B^n(x, r) \subset G$, and we call $r(x, f)$ the radius of injectivity of f at x. If $G = B^n = B^n(0, 1)$ and $x = 0$, we abbreviate $r(f) = r(0, f)$.

Martio, Rickman and Väisälä proved in [MRV3; 2.3] that if $f : B^n \to R^n$ is a K-quasiregular local homeomorphism and $n \geq 3$, then $r(f) \geq c > 0$, where c is a constant depending only on n and $K \geq 1$. In other words, if $n \geq 3$,

$$\psi_n(K) = \inf_f r(f) > 0, \tag{1.2}$$

where the infimum is taken over all K-quasiregular local homeomorphisms $f : B^n \to R^n$. If $n = 2$, (1.2) fails; for instance, the mappings $f_j : B^2 \to R^2$, $f_j(z) = e^{jz}$, $j = 1, 2, \cdots$, provide a counter-example.

Let $f : G \to R^n$, $n \geq 3$, be K-quasiregular and let B_f denote the branch set of f. If $x \in G \setminus B_f$, it is easy to see by similarity mappings that

$$r(x, f) \geq \psi_n(K) d(x, B_f \cup \partial G) > 0, \tag{1.3}$$

where ∂G is the boundary of G (taken in $\bar{R}^n$) and $d(x, A)$ is the euclidean distance from x to a non-empty set $A \subset \bar{R}^n$. Due to (1.3) we call $\psi_n(K)$ the *coefficient of injectivity* for quasiregular mappings. Furthermore, (1.3) immediately implies the theorem of Zorič [Z]: If $f : R^n \to R^n$ is a quasiregular local homeomorphism and $n \geq 3$, then f is a homeomorphism.

In this paper we will study the behavior of the function $\psi_n : [1, \infty) \to (0, 1]$ defined by (1.2) for $n \geq 3$. Our main results are:
(i) If $t \geq s \geq 1$, then

$$\exp(-a_n t^{1/(n-1)}) \leq \psi_n(t) \leq \exp(-\alpha_n(s) t^{1/(n-1)}),$$

where

$$a_n \in (0, \infty) \quad \text{and} \quad \lim_{s \to \infty} \alpha_n(s) = a_n ,$$

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