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A p-ADIC THEORY OF HECKE POLYNOMIALS

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1. Introduction.

Let p be a fixed odd prime and let $\bar{\mathbf{F}}_p$ be the algebraic closure of $\mathbf{F}_p$, the prime field of characteristic p. Denote by E_λ the elliptic curve whose projective equation is

$$y^2z = x(x - z)(x - \lambda z), \tag{1.1}$$

where $\lambda \in \bar{\mathbf{F}}_p$ and $\lambda \neq 0, 1$. The zeta function of E_λ is defined by

$$\zeta(E_\lambda, t) = \exp\left(\sum_{m=1}^{\infty} N_m t^m/m\right), \tag{1.2}$$

where N_m is the number of points of E_λ rational over $\mathbf{F}_q$, the finite field of $q = p^{m(\deg\lambda)}$ elements ($\deg \lambda = [\mathbf{F}_p(\lambda) : \mathbf{F}_p]$). It is well known that the zeta function has the form

$$\zeta(E_\lambda, t) = \frac{(1 - \pi_1(\lambda)t)(1 - \pi_2(\lambda)t)}{(1 - t)(1 - tp^{\deg\lambda})}. \tag{1.3}$$

Let $\bar{S}'$ be the set of supersingular moduli, i.e., the zeros of the Hasse invariant

$$g(\lambda) = \sum_{j=0}^{(p-1)/2} ((\tfrac{1}{2})_j/j!)^2\lambda^j, \tag{1.4}$$

and let $\bar{S}$ be the union of $\bar{S}'$ with the set $\{0, 1, \infty\}$. In [8], Dwork gives a p-adic theory for the infinite product defined for each positive integer k by

$$M_{k+2}(t) = \prod_{\lambda} \prod_{j=0}^{k} (1 - t^{\deg\lambda}\pi_1(\lambda)^{k-j}\pi_2(\lambda)^j)^{-1/\deg\lambda}, \tag{1.5}$$

where the outer product is over the complement of $\bar{S}$ in $\bar{\mathbf{F}}_p$. More precisely, Dwork shows that

$$M_{k+2}(t) = \det(I - t\alpha \mid L/l_kL),$$

where $\det(I - t\alpha \mid L/l_kL)$ is the Fredholm determinant of a completely continuous endomorphism α of a p-adic Banach space L/l_kL. From this one can deduce that $M_{k+2}(t)$ is actually a polynomial. The Banach space L/l_kL is the cokernel of a linear differential operator l_k, acting on the space L of functions

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