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INTEGRAL REPRESENTATION FORMULAS ON STRICTLY PSEUDOCONVEX DOMAINS IN STEIN MANIFOLDS

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Introduction.

The integral formula of Cauchy holds a central position in complex analysis of one variable and its direct generalization to polydiscs is important in establishing the elementary properties of analytic functions of several complex variables, e.g., the nonexistence of isolated singularities in $\mathbf{C}^n$, $n \geq 2$. In the 1930's Bergman and Weil [2, 22] developed a generalization of the Cauchy formula for analytic polyhedra, and in the early 1940's Bochner and Martinelli [3, 15] produced a general integral formula for functions holomorphic on piecewise-smoothly bounded domains in $\mathbf{C}^n$. Later in 1961 Norguet [17] proved a general integral formula (based on the Cauchy-Fantappiè kernel introduced by Leray [13]) which included the above formulas as special cases. Finally in 1970 Ramírez [19] and Henkin [9, 10] gave integral representations with holomorphic kernels on strictly pseudoconvex domains in $\mathbf{C}^n$. These allowed Grauert-Lieb [7] and Henkin [10] to solve the $\bar{\partial}$-equation for (0, 1)-forms with uniform bounds on strictly pseudoconvex domains in $\mathbf{C}^n$. That is, if D is a strictly pseudoconvex domain in $\mathbf{C}^n$ with smooth boundary and if f is a uniformly bounded and $\bar{\partial}$-closed, C^∞, (0, 1)-form on D, then there exists a uniformly bounded C^∞ function u on D with $\bar{\partial} u = f$. Others extended these results to $(0, q)$-forms and L^p and Hölder estimates. Under a local version of the Grauert-Lieb method, Kerzman [12] generalized these results to strictly pseudoconvex domains with smooth boundary in Stein manifolds. More recently Stout [21] has proved an integral formula for holomorphic functions on a strictly pseudoconvex domain in a codimension-one submanifold of $\mathbf{C}^n$.

In this paper we derive integral formulas for $\bar{\partial}$-closed, C^∞, $(0, q)$-forms on a strictly pseudoconvex domain D in a Stein manifold such that the holomorphic tangent bundle of the manifold is stably trivial over a neighborhood of $\bar{D}$. Using these formulas we then solve the $\bar{\partial}$-equation with uniform bounds for $(0, q)$-forms on such domains D. This is a special case of the results of Kerzman, but our method appears to be more direct since it does not require the patching together of local solutions.

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