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CONSTRUCTING STRANGE MANIFOLDS WITH THE DODECAHEDRAL SPACE

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Since its discovery by Poincaré at the beginning of the century, the dodecahedral manifold K has fueled several of the most fundamental theorems on manifolds. Originally K was presented as an example of a homology 3-sphere which is not a sphere, providing a counterexample to the "homology" Poincaré conjecture [ST]. Milnor's original counterexample Σ to the smooth Poincaré conjecture is closely connected to K in the theory of complex singularities [Mi_1], [Mi_3]. Since Σ is a PL sphere, it provided the first example of a PL manifold with more than one smooth structure, thus distinguishing between the categories $DIFF$ and PL.

Most recently the manifold K has been used by Kirby and Siebenmann to distinguish between PL manifolds and merely topological manifolds. In particular, their fundamental example of a non-PL manifold is a 5-manifold M homotopy equivalent to $X \times S^1$, where X is a homology 4-manifold whose only non-Euclidean point has link K [Sie].

In this paper three manifolds are constructed which are closely related to the results of Kirby–Siebenmann. Their existence has been demonstrated elsewhere [Sh_1], [CS], [HoM]. Here the constructions flow from properties of K. §1 presents two of the many descriptions of K. In §2 fake homotopy structures are constructed for $S^3 \times S^1 \# S^2 \times S^2$ and $S^3 \times S^1 \times S^1$. In §3 a nontriangulable 5-manifold homotopy equivalent to $CP(2) \times S^1$ is constructed, using deep results of Kirby–Siebenmann.

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§1. The dodecahedral manifold K.

The dodecahedral space is remarkable not only for its colorful past, but also for the multitude of ways in which it can be defined. Originally it was constructed by identifying opposite sides of the dodecahedron [ST]. It is also the p-fold branched cyclic covering of a torus knot of type (q, r), where (p, q, r) is any permutation of $(2, 3, 5)$. It can be constructed by plumbing 2-disk bundles of euler characteristic 2 over S^2 along a tree with branches of length 2, 3 and 5.

The descriptions which will be of most interest here are the following. First, K is the intersection of the unit 5-sphere in $\mathbf{C}^3$ with the complex variety $z_0^2 +$

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