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ON THE INDEX OF P-ADIC DIFFERENTIAL OPERATORS II

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Introduction.

In a recent article [6] we proved the following result. Let L be a linear differential operator with polynomial coefficients (over a p-adic field K). If the homogeneous equation $Lu = 0$ has no solution analytic in the generic disk (we shall say that L has a 0-kernel in the generic disk), then L, as linear operator over the space $W_a^{\,r}$ of functions analytic in a disk $D(a, r^-)$ with a given growth at the boundary, is injective and has an index, provided r is big enough. Moreover we gave an estimate of the index ([6], theorem 4.16).

It was also shown that the hypothesis that L has a 0-kernel in the generic disk cannot be weakened; if there is a u analytic in the generic disk, such that $Lu = 0$, then L has no index in the space of bounded analytic functions in that disk (cf. theorem 2.10 below).

In this article we shall consider the space $H(A)$ of analytic elements over the set A, i.e., the space of uniform limits on A of rational functions without singularities in A. We shall prove that under the same hypothesis on L (that is L has a 0-kernel in the generic disk), L, as linear operator over $H(A)$, is injective and has an index. provided the distance between A and $\complement A$ ($\complement$ is the symbol for "complement of") is big enough. (See Theorem 3.4 for a precise statement.) Observe that if A is the disk $D(a, r)$, then the distance between A and $\complement A$ is precisely r, so our condition is the natural generalization of the condition mentioned in the first paragraph.

The difference with [6] is that we consider a much wider class of sets A over which our analytic functions are defined (in [6] we considered only disks). On the other hand we have more restrictions on the growth of the functions (in fact analytic elements on A are bounded functions). Also in [6], we considered only operators with polynomial coefficients. This was an unnecessary restriction and here we take operators with analytic coefficients. In fact the proofs given for polynomial coefficients were actually valid for analytic coefficients and in the applications (see [3] for example) we do need to consider the case of analytic coefficients.

An important consequence of an operator's having an index (apart from the fact that this gives information about the existence and uniqueness of the solutions of the non-homogeneous equation) is that this gives results on the regularity ([16] Theorem 4.16) and the analytic continuation (Corollaries 3.5 and 3.6 below) of these solutions.

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