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A PATH LIFTING CONSTRUCTION FOR DISCRETE OPEN MAPPINGS WITH APPLICATION TO QUASIMEROMORPHIC MAPPINGS

SEPPO RICKMAN

1. Introduction. The purpose of this paper is to show that given a discrete open mapping and a sphere separating two points omitted by the mapping there exists a family of paths lying in the sphere which has a significant lower bound for its modulus in terms of multiplicity and whose paths lift to "long" paths. The motivation for such a construction arises from the theory of quasiregular mappings where it can be used to estimate the growth of multiplicities. Recently O. Martio [2] applied the construction to prove that a k-periodic quasiregular mapping of the euclidean n-space R^n into itself cannot have a finite multiplicity in a period strip if $k \leq n - 2$.

2. Notation and terminology. We shall mostly use the same notation and terminology as in [3, 4]. For $x \in R^n$ we write $x = x_1e_1 + \cdots + x_ne_n$ where $e_1, \cdots, e_n$ is the standard orthonormal basis in R^n. For a set $A \subset R^n$ the closure $\bar{A}$, the boundary ∂A, and the complement $\complement A$ are all taken with respect to $\bar{R}^n = R^n \cup \{\infty\}$. The spherical (chordal) metric in $\bar{R}^n$ is denoted by q. The inner product of $x, y \in R^n$ is $(x \mid y)$. For $x \in R^n$ and $r > 0$ we set

$$B^n(x, r) = \{y \in R^n \mid |y - x| < r\},$$
$$S^{n-1}(x, r) = \partial B^n(x, r),$$
$$B^n(r) = B^n(0, r), \quad S^{n-1}(r) = S^{n-1}(0, r).$$

By ω_{n-1} we denote the $(n - 1)$-dimensional measure of $S^{n-1}(1)$.

Let $n \geq 2$ and let $f : G \to \bar{R}^n$ be a continuous map of a domain G in R^n. If $A \subset G$, $y \in \bar{R}^n$, and $B \subset \bar{R}^n$, we define the following multiplicities (possibly ∞):

$$N(y, f, A) = \operatorname{card} f^{-1}(y) \cap A,$$
$$N(B, f, A) = \sup_{y \in B} N(y, f, A),$$
$$N(f, A) = N(\bar{R}^n, f, A).$$

If f is discrete and open, every $x \in G$ has arbitrarily small normal neighborhoods U, i.e. $\bar{U} \subset G$, $f\partial U = \partial fU$, and $U \cap f^{-1}(f(x)) = \{x\}$. Such neighborhoods can be chosen to be the x-component $U(x, f, r)$ of $f^{-1}B^n(f(x), r)$ for small $r > 0$ if $f(x) \neq \infty$ [3, 2.9].

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