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MATRIX MODELS FOR OPERATORS

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Let σ, G_+, G_- be complex sets such that σ is compact, G_+ and G_- are open and

$$G_+ \cap G_- = \phi, \qquad G_+ \cup G_- \subset \sigma.$$

Let N denote the set of all natural numbers and let

$$\varphi_+ : G_+ \to N, \qquad \varphi_- : G_- \to N$$

be continuous functions. The aim of this note is to produce canonical forms modulo compact perturbations, for operators T, acting in a separable Hilbert space, which fulfill the conditions

$$\sigma = \sigma_W(T) \text{ (= the Weyl spectrum)}$$

$$\sigma \backslash (G_+ \cup G_-) = \sigma_e(T) \text{ (= the essential spectrum)}$$

$$\text{ind } (T - \lambda) = \begin{cases} \varphi_+(\lambda), & \lambda \in G_+ , \\ -\varphi_-(\lambda), & \lambda \in G_- . \end{cases}$$

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Throughout, we shall denote by H a separable Hilbert space over the complex field Λ. The algebra of all bounded linear operators acting in H will be denoted by $\mathcal{L}(H)$, and $\mathcal{K}(H)$ will be the ideal of all compact operators acting in H.

Let $T \in \mathcal{L}(H)$ and denote by $\tilde{T}$ the image of T in the Calkin algebra. The set $\sigma(\tilde{T})$ will be called the essential spectrum of T, denoted $\sigma_e(T)$, and its complement is the Fredholm domain of T, denoted $\rho_F(T)$. For any $\lambda \in \rho_F(T)$ we have

$$\max\{\dim \ker(T - \lambda), \dim \ker(T - \lambda)^*\} < \infty,$$

thus the index function

$$\lambda \to \text{ind}(T - \lambda) = \dim \ker(T - \lambda) - \dim \ker(T - \lambda)^*, \qquad \lambda \in \rho_F(T),$$

is well defined. If we put

$$\rho_F^{\,+}(T) = \{\lambda \in \rho_F(T) : \text{ind } (T - \lambda) > 0\},$$

$$\rho_F^{\,-}(T) = \{\lambda \in \rho_F(T) : \text{ind } (T - \lambda) < 0\},$$

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