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RANDOMLY CONTINUOUS FUNCTIONS AND SIDON SETS

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1. Introduction. Drury [1] has shown that for compact abelian groups the union of two Sidon sets is also a Sidon set. The proof depends strongly on the use of Riesz products. In this paper a proof of Drury's theorem is given which uses Riesz products only for the set of projections of the infinite torus. It is part of a more general theorem which states that "continuous" can be replaced by "randomly continuous" in the definition of Sidon set.

Let G be a compact abelian group with dual group Γ. For a function $f \in L_1(G)$ or a measure $\mu \in M(G)$ the Fourier transform will be denoted by $\hat{f}$(resp. $\hat{\mu}$). To G we associate the infinite torus T^∞ indexed by Γ; that is $T^\infty = \prod_{\gamma \in \Gamma} T_\gamma$ where each T_γ is the circle group. φ_γ will denote the projection of T^∞ onto T_γ.

For $f \in L_1(G)$ and $t \in T^\infty$ we consider the formal Fourier series on G

$$\sum_{\Gamma} \hat{f}(\gamma)\varphi_\gamma(t)\gamma(x) \qquad (x \in G). \tag{1}$$

f is said to be *randomly continuous* if, for almost all $t \in T^\infty$, (1) is the Fourier series of a continuous function. Necessary conditions and sufficient conditions for f to be randomly continuous (at least on the circle group) have been given by Salem and Zygmund [6] (see also [2], [3]).

Let $C(G)$ and $R(G)$ be the spaces of continuous and randomly continuous functions on G and let $A(G)$ be the space of functions with absolutely convergent Fourier series. For the sake of a neat diagram we also define $C'(G)$ as those $f \in L_1(G)$ such that (1) is the Fourier series of a continuous function for at least one $t \in T^\infty$. Then

$$A(G) \subset R(G) \subset C'(G) \subset L_2(G). \tag{2}$$

If $B \subset L_1(G)$ and $E \subset \Gamma$ then B_E will denote those $f \in B$ such that $\hat{f}$ is supported by E. A set E is called a *Sidon set* if $C_E = A_E$. Clearly this is the same as saying $C'_E = A_E$.

THEOREM 1. *If $R_E = A_E$ then $C'_E = A_E$.*

This says that E is Sidon if and only if every randomly continuous E function has an absolutely convergent Fourier series.

The problem with considering the union of Sidon sets is that if $f \in C(G)$ and $E \subset \Gamma$ then it is not always true that $\sum_E \hat{f}(\gamma)\gamma \in C(G)$. However it is

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