Vol. 42, No. 4 DUKE MATHEMATICAL JOURNAL© December 1975

RESTRICTIONS OF H^p FUNCTIONS TO THE DIAGONAL OF THE POLYDISC

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A function $f(z) = f(z_1, \cdots, z_n)$ analytic in the polydisc U^n is said to be of class $H^p(U^n)$, $0 < p < \infty$, if the integrals

$$\int_{T^n} |f(r\zeta)|^p \, dm_n(\zeta), \qquad 0 < r < 1,$$

are bounded, where T^n is the torus and m_n is the normalized Lebesgue measure on T^n. The Bergman space A^p consists of all functions f analytic in the unit disc for which the area integral

$$\int_0^{2\pi} \int_0^1 |f(re^{i\theta})|^p \, r \, dr \, d\theta < \infty .$$

For $n = 2$ and for $p = 1, 2$, Walter Rudin [3, pp. 53, 69] showed that if $f \in H^p(U^n)$, then its restriction $g(w) = f(w, w, \cdots, w)$ to the diagonal of U^n belongs to A^p; and he posed the problem to extend these results to other values of p and n, and to other subvarieties in place of the diagonal. The following theorem constitutes a partial solution to Rudin's problem.

THEOREM 1. *Suppose $0 < p \le q < \infty$ and $n \ge 2$. Let $f \in H^p(U^n)$, and let g be the restriction of f to the diagonal. Then*

$$\int_0^{2\pi} \int_0^1 |g(re^{i\theta})|^q (1 - r)^{nq/p-2} r \, dr \, d\theta < \infty .$$

COROLLARY. *If $f \in H^p(U^2)$, then $g \in A^p$, $0 < p < \infty$.*

Since $|f(z)|^\lambda$ is n-subharmonic for each $\lambda > 0$, a trivial estimate of the Poisson kernel shows that

$$|g(are^{i\theta})|^p \le C(1 - r)^{-n}, \qquad 0 < r < 1.$$

Thus it suffices to prove the theorem for $q = p$, in which case it is an immediate consequence of the following more general theorem.

THEOREM 2. *Suppose $2 \le p < \infty$ and $n \ge 2$. Then for some constant C and for all nonnegative n-subharmonic functions u in U^n,*

$$\int_0^{2\pi} \int_0^1 [u(re^{i\theta}, \cdots, re^{i\theta})]^p (1 - r)^{n-2} r \, dr \, d\theta \le C \lim_{r \to 1} \int_{T^n} [u(r\zeta)]^p \, dm_n(\zeta).$$

Received July 22, 1974. Revision received June 16, 1975.