Vol. 42, No. 4 DUKE MATHEMATICAL JOURNAL© December 1975

CLASSICAL AND NON-CLASSICAL SCHOTTKY GROUPS

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§0. Introduction.

In this paper we include several results about classical and non-classical Schottky groups. We show that the space of marked classical Schottky groups is connected. Also, we produce an example of a non-classical Schottky group of rank 2, as well as give conditions ensuring that a Schottky group of rank 2 be non-classical.

§1. Preliminaries.

Let $C_1, D_1, \cdots, C_n, D_n$ be a collection of $2n$ mutually disjoint Jordan curves in the extended complex plane $\hat{C}$ which bound a $2n$ connected region Ω, and suppose that $T_1, \cdots, T_n$ is a set of n moebius transformations with the property that

(1) $T_i(C_i) = D_i$ and,

(2) $T_i(\Omega) \cap \Omega = \varnothing$, $i = 1, \cdots, n$.

The group G, generated by $T_1, \cdots, T_n$, is called a Schottky group, and $T_1, \cdots, T_n$ is called a set of standard generators. It is not hard to see that G is a free, purely loxodromic (here this term includes hyperbolic), discontinuous group. Conversely, Maskit [3] has shown that every free, finitely generated, purely loxodromic discontinuous group is a Schottky group.

If in the definition of a Schottky group we require that the Jordan curves be circles, (this term includes straight lines), then the resulting group is called a classical Schottky group. A non-constructive proof of the existence of non-classical Schottky groups was given by Marden [2].

A marked Schottky group is a Schottky group together with a choice of standard generators. Chuckrow [1] shows that every set of free generators is standard. She also shows that the collection of all marked Schottky groups can be embedded naturally as an open, connected subset of a manifold of dimension $3n$. The resulting topology is the same as that obtained by the convergence of the matrices (in PSL $(2, \mathbf{C})$) determined by the generators. Thus if $G_m = \langle T_{1m}, \cdots, T_{nm} \rangle$, $m = 0, 1, 2, \cdots$, is a sequence of marked Schottky groups and T_{im} determines that element of PSL $(2, \mathbf{C})$ described by the matrix $\begin{bmatrix} a_{im} & b_{im} \\ c_{im} & d_{im} \end{bmatrix}$, where $a_{im} d_{im} - b_{im} c_{im} = 1$, then $G_m \to G_0$ if and only if

$$\begin{bmatrix} a_{im} & b_{im} \\ c_{im} & d_{im} \end{bmatrix} \to \begin{bmatrix} a_{i0} & b_{i0} \\ c_{i0} & d_{i0} \end{bmatrix}$$

Received March 22, 1975. Revision received June 27, 1975.