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MICROLOCAL PARAMETRICES FOR DIFFRACTIVE BOUNDARY VALUE PROBLEMS

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Using the theory of oscillatory integrals (Fourier integral operators) it has been possible for some time to construct parametrices for differential operators with real principal symbol and simple characteristics (see Lax [10], Duistermaat and Hörmander [3]). These constructions depend, at some stage, on the solution of an associated Cauchy problem. So it is not surprising that general Dirichlet boundary value problems for such operators can be treated by a combination of this method and the theory of elliptic boundary value problems, provided the extra condition that the null bicharacteristic directions of the operator be nowhere tangent to the boundary is added (see Nirenberg [12]).

Classical existence and uniqueness theorems for hyperbolic mixed problems are insensitive to such transversality conditions, so it is to be expected that parametrices can still be constructed despite the occurrence of 'glancing points', where the bicharacteristic directions are tangent to the boundary.

The main problem that occurs in such a construction, at least if it is based on oscillatory integral methods, is that the canonical relation defining the flow out of singularities from the boundary is singular and hence, so is its generating function. In [11] a class of operators designed to overcome this problem was developed and these will be used here to construct microlocal parametrices for the Dirichlet problem for second order operators near one of the two simplest situations in which there are glancing points. This is described as 'diffraction' since it occurs in the classical problem of scattering by a convex body and it is associated with the appearance of shadows, that is bicharacteristically inaccessible regions. Indeed, the constructions below were motivated by the analysis of one such problem carried out by Friedlander [4].

The plan of the paper is as follows. Section 1 contains a discussion of parametrices for Dirichlet boundary value problems for a second-order operator and in section 2 glancing points are classified in terms of the local behaviour of the bicharacteristics of the operator. The geometry of the diffractive situation is considered further in section 3 leading to the statement of a theorem on the propagation of singularities for diffractive problems proving the existence of the 'shadow regions'. The main results on the existence of parametrices are stated in section 4 and proved in sections 10 and 11 after the preliminary constructions, of phase functions and symbols, in sections 5 to 9.

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