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LOCAL FOURIER-AIRY INTEGRAL OPERATORS

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In this paper a new class of operators defined by oscillatory integrals is introduced. These operators, here called Fourier-Airy integral operators, will be used ([6]) in the construction of parametrices for boundary value problems. In that construction a Fourier-Airy intergal operator gives the major contribution to a microlocal parametrix near each non-degenerate diffractive point on the boundary. Such diffractive points are associated with the degeneracy of the phase functions used in the standard construction of parametrices (see for example [1], [4], [7]) and this degeneracy is introduced through the Airy function. Thus there is a hypersurface on one side of which the (homogeneous) phase function is real whilst on the other side it is complex. As opposed to the situation considered in [5] this homogeneous phase function is singular at the surface.

The operators discussed below are maps

$$B : \mathfrak{D}'(\mathbf{R}^N) \to C^\infty([0, \infty); \mathfrak{D}'(\mathbf{R}^N))$$

where, formally,

$$(0.1) \qquad [B(f)](x; y) = \frac{1}{(2\pi)^N} \int \exp\,(i\varphi(x, y, \xi) - i\varphi(0, y', \xi))f(y')\sigma(\zeta(0, y', \xi))$$
$$\times \frac{a(x, y, y', \xi)A(\zeta(x, y, \xi)) + b(x, y, y', \xi)A'(\zeta(x, y, \xi))}{A(\zeta(0, y', \xi))}\, d\xi\, dy'.$$

In (0.1) φ, the principal phase function is real, homogeneous of degree one in the ξ variable and satisfies certain non-degeneracy conditions. A is an Airy function. For negative values of its argument $A(\zeta)$ behaves like $\exp(\pm 2i(-\zeta)^{\frac{3}{2}}/3)$ so ζ, the secondary phase function, is homogeneous of degree two-thirds. When its argument is positive A is exponentially increasing so the cutoff factor σ, with $\sigma(s) = 0$ if $s \geq s_0 > 0$, is introduced to make the integral converge, at least in the sense of oscillatory integrals. Finally, a and b are symbols. The operator has two symbols because the Airy function and its first derivative, A', are linearly independent so both must be included to make the class of Fourier-Airy operators manifestly invariant under composition on the left with differential operators.

Section one contains a brief summary of the relevant properties of Airy

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