Vol. 42, No. 3 DUKE MATHEMATICAL JOURNAL© September 1975

EQUICONTINUITY THEOREM WITH AN APPLICATION TO VARIATIONAL INTEGRALS

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1. Introduction

Let $G \subset \bar{R}^n$ be a domain and $x \in \partial G$. We say that $M_x < \infty$ if there exists a non-degenerate continuum $C \subset G \cup \{x\}$ such that $x \in C$ and the n-modulus of the path family joining C and ∂G in G is finite. We prove, in Theorem 4.6, that a family $\mathfrak{M}$ of continuous, monotone functions $u : \bar{G} \to R$ with uniformly bounded n-Dirichlet integral

$$\int_{G \cap R^n} |\nabla u|^n \, dm$$

is equicontinuous on $\bar{G}$ if $\mathfrak{M} \mid \partial G$ is equicontinuous and if for each point $x \in \partial G$ the condition $M_x < \infty$ is not satisfied. This theorem is a generalization of the results in [7] and in [5]. In [7, 4.3.4] this fact was proved for Lipschitz-domains and in [5] for quasiconformally collared domains. Theorem 4.6 seems to be new even for $n = 2$.

In chapter 5 we apply the above result to solve the Dirichlet problem associated with a wide class of kernels $F(x, u(x), \nabla u(x))$ in "the borderline case" $F(x, p, q) \approx |q|^n$.

2. Preliminaries

2.1. *Notation.* The two point compactification of R is denoted by $\dot{R}$. We let R^n denote the euclidean n-space with the usual norm $| \ |$ and for $x \in R^n$ we write $x = (x_1, \cdots, x_n) = x_1 e_1 + \cdots + x_n e_n$ where $e_1, \cdots, e_n$ are coordinate unit vectors of R^n. $\bar{R}^n$ means the one point ∞ compactification of R^n. For each set $A \subset \bar{R}^n$ we let $\complement A$, $\bar{A}$, ∂A, and int A denote the complement, closure, boundary, and interior of A, all taken with respect to $\bar{R}^n$. $d(A)$ is the euclidean diameter of A. Given two sets A and B in $\bar{R}^n$, $A \backslash B$ is the set theoretic difference of A and B and $d(A, B)$ the euclidean distance of A and B. A continuum in $\bar{R}^n$ is a compact connected set which contains more than a point. Given $x \in R^n$ and $r > 0$, we let $B^n(x, r)$ denote the open ball $\{y \in R^n : |y - x| < r\}$ and $S^{n-1}(x, r) = \partial B^n(x, r)$. We shall also employ the abbreviations $B^n(r) = B^n(0, r)$, $B^n = B^n(1)$, $S^{n-1}(r) = S^{n-1}(0, r)$, and $S^{n-1} = S^{n-1}(1)$.

The Lebesgue measure of a set $A \subset R^n$ will be written as $m(A)$. ω_{n-1} means the $(n - 1)$-measure of S^{n-1}.

Received March 18, 1975.