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DEFORMATIONS OF THE SCALAR CURVATURE

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§0. Introduction and Statement of Results.

Our main goal is to study the relationship between infinitesimal and actual perturbations of the scalar curvature function with respect to a varying metric and to describe when solutions of the linearized equations can be used to approximate solutions of the nonlinear ones. Many of the results are motivated by corresponding questions in general relativity [17]. This paper supplies details and extensions of the results announced in [19] and described in [20]. The starting point of our proofs is the basic work of Ebin [14] and Berger-Ebin [4]. We begin with a few general notions.

DEFINITION. *Let X and Y be topological vector spaces and $F : X \to Y$ a differentiable mapping. We say F is linearization stable at $x_0 \in X$ iff for every $h \in X$ such that $DF(x_0)\cdot h = 0$ there exists a differentiable curve $x(t) \in X$ with $x(0) = x_0$, $F(x(t)) = F(x_0)$ and $x'(0) = h$.*

The implicit function theorem gives a simple criterion for linearization stability as follows.

CRITERION. *Let X, Y be Banach spaces, let $F : X \to Y$ be C^1 and suppose $DF(x_0)\colon X \to Y$ is surjective and its kernel splits, i.e. F is a submersion at x_0. Then F is linearization stable at x_0.*

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