Vol. 42, No. 3 DUKE MATHEMATICAL JOURNAL© September 1975

SOME INEQUALITIES FOR THE SPECTRAL RADIUS OF NON-NEGATIVE MATRICES AND APPLICATIONS

by

S. FRIEDLAND AND S. KARLIN

Contents

1. Introduction. The purpose of this work is to establish useful lower and upper estimates for the spectral radius of certain classes of positive matrices which apart from their independent interest are pertinent to the study of a number of mathematical models of population genetics and also apply to the solution of some cases of inverse eigenvalue problems.

In the stability analysis of certain equilibria states of physical and biological systems, it is relevant to determine useful conditions indicating when the largest eigenvalue ρ for a matrix of the type MD (composed from a general non-negative and positive diagonal matrix) exceeds or is smaller than 1. In the physical setting, $M = ||m_{ij}||$ is commonly an $n \times n$ matrix of non-negative elements corresponding to a Green's function for a vibrating coupled mechanical system of n mass points, while D is a diagonal matrix with positive diagonal entries $\{d_1, d_2, \cdots, d_n\}$ such that d_i, $i = 1, 2, \cdots, n$, relates to the mass at position i.

In the genetics context, a population is distributed in n demes (habitats, $\{\mathcal{P}_1, \mathcal{P}_2, \cdots, \mathcal{P}_n\}$) subject to local natural selection forces and inter-deme migration pressures. The changes in the population composition of a trait expressed by two possible types (genes) labeled **A** and **a** are observed over

Received March 13, 1975. The first author supported in part by NSF-MPS72-05055-A02; second author supported in part by NSF-MPS71-02905-A03.