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EQUIVARIANT BORDISM EXACT SEQUENCES

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§1. Introduction. In (8) C.T.C. Wall shows that there are exact sequences

$$\Omega_* \to \Omega_* \to W_* \tag{1}$$

$$0 \to W_* \to N_* \to N_* \to 0 \tag{2}$$

relating oriented bordism, Ω_*, unoriented bordism, N_*, and Wall bordism, W_*, where the maps of the sequences are defined geometrically. (1) and (2) splice together to give an exact triangle

$$\Omega_* \otimes N_* \to \Omega_* \to N_* . \tag{3}$$

In [10] algebraic techniques are used to show that an equivariant analogue of (3) exists for groups G having odd order, but the geometry of the situation is lost. In this paper equivariant Wall manifolds W_*^G are defined and geometric arguments are used to show that for all finite supersolvable groups G there is an equivariant version of (1) and that for G supersolvable of odd order both (1) and (2) have G analogues. In the last section of this paper the question of the exactness of the equivariant Rohlin sequence is completely answered for finite groups.

§2. Some preliminaries.

(a) *A classifying space for equivariant line bundles.* If V is a finite dimensional representation space for a finite group G, then $V = V_1 \oplus \cdots \oplus V_k$ where each V_q is a sum of v_q copies of the q_{th} distinct irreducible real representation of G. One defines a partial ordering on the collection of G representations by defining $V \leq W$ if $v_q \leq w_q$ for each q. For $V \leq W$ there is a G map from V into W which identifies V_q with the first v_q summands of W_q. This ordering and these maps induce a partial ordering and maps on the collection of projective spaces of G representations. Further, over the G projective space $P(V)$ lives the canonical G line bundle λ_V and if $g : P(V) \to P(W)$ is the map defined above, $g^*(\lambda_W) = \lambda_V$. Hence one has a directed system of G-spaces and G line bundles, and if one takes the limit over this system, one gets the classification space for G line bundles, together with its canonical line bundle. (see [1; §1.6]).

(b) *Equivariant transverse regularity.* To proceed with the geometric analysis of the Wall sequences one needs some equivariant versions of transverse regularity. Let G be a finite supersolvable group.

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