Vol. 42, No. 3 DUKE MATHEMATICAL JOURNAL© September 1975

REGULAR AND SINGULAR PERTURBATION PROBLEMS FOR A SINGULAR ABSTRACT CAUCHY PROBLEM

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1. Introduction. Regular and singular perturbations have received extensive treatment in the literature for problems involving operator differential equations with coefficients satisfying certain regularity conditions. Usually the coefficients are constants or they satisfy a boundedness condition. We mention that Bobisud and Calvert [1], Bobisud and Hersh [2], and Friedman [6] have obtained very general results for these cases. The work of Friedman, and Bobisud and Calvert as well as earlier work of Latil [12], Kisynski [11], and Smoller [15] employed the spectral representation of the operators involved in the equations. Bobisud and Hersh, Schoene [14], and Griego and Hersh [7] have investigated equations involving non-self-adjoint operators. Schoene considered $\epsilon u_\epsilon''(t) + u_\epsilon'(t) = A^2 u_\epsilon(t)$, where A generates a group. He obtained explicit representations of u_ϵ and demonstrated convergence in $O(\epsilon)$ for sufficiently regular initial data. Griego and Hersh obtained a weaker convergence theorem for this equation by employing probabilistic methods.

In the present paper we use the formula

$$u(t) = \int_{-1}^{1} f(k, s) G_A(st)\phi \, ds, \qquad k > 0, \tag{1.1}$$

where

$$f(k, s) = C(k)(1 - s^2)^{k-1}$$

and

$$C(k) = \Gamma(k + (\tfrac{1}{2}))/[\Gamma(k)\Gamma(\tfrac{1}{2})]$$

to investigate two types of perturbation problems involving a singular abstract Cauchy problem. The integral operator in (1.1) is the transmutation operator of Delsarte which can be found in the works of Lions, who calls it the Poisson operator and refers also to Delsarte. This operator plays the same role in our investigation as the Riemann functions play in Hadamard's work [8, pp. 103–104] where the solution of the heat equation is obtained as the limit of solutions of an associated hyperbolic equation with a small parameter.

We now state certain basic results from the theory of bounded groups of linear operators and formulate the class of singular abstract Cauchy problems which will receive primary attention here.

Received December 2, 1974. Revision received April 9, 1975. Supported in part by NSF Grant No. GY-11066.