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THE REDUCING ESSENTIAL MATRICIAL SPECTRA OF AN OPERATOR

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1. Introduction.

In this paper we continue our study of the reducing matricial spectra and the reducing essential matricial spectra of a (bounded, linear) operator initiated in [10].

Let $\mathcal{H}$ be a separable infinite dimensional complex Hilbert space and let $\mathcal{L}(\mathcal{H})$ denote the algebra of all operators on $\mathcal{H}$. The ideal of all compact operators in $\mathcal{L}(\mathcal{H})$ will be denoted by $\mathbf{K}(\mathcal{H})$ and the canonical quotient map from $\mathcal{L}(\mathcal{H})$ onto the (Calkin) algebra $\mathcal{L}(\mathcal{H})/\mathbf{K}(\mathcal{H})$ will be denoted by π.

If $\mathfrak{A}$ is a C^*-algebra and n is a positive integer, then an *n-dimensional representation* φ of $\mathfrak{A}$ is a *-algebra homomorphism from $\mathfrak{A}$ into the C^*-algebra $\mathbf{M}_n$ of all $n \times n$ complex matrices. Such a representation φ will be called *irreducible* if $\varphi(\mathfrak{A}) = \mathbf{M}_n$ and will be called *non-degenerate* if the identity of the C^*-algebra $\varphi(\mathfrak{A})$ coincides with the identity matrix 1_n of $\mathbf{M}_n$. Given an operator T in $\mathcal{L}(\mathcal{H})$ we shall denote by $\mathcal{C}^*(T)$ the C^*-algebra generated by T and $1_{\mathcal{H}}$. Also, the C^*-algebra $\pi(\mathcal{C}^*(T))$ which is clearly the C^*-algebra generated by $\pi(T)$ and $\pi(1_{\mathcal{H}})$ will be denoted by $\mathcal{C}_e^*(T)$.

Following [10], for every operator T in $\mathcal{L}(\mathcal{H})$ and every positive integer n we define the *reducing $n \times n$ spectrum* of T to be the set $R^n(T)$ consisting of all those matrices L in $\mathbf{M}_n$ for which there exists a non-degenerate (but not necessarily irreducible) n-dimensional representation φ of $\mathcal{C}^*(T)$ such that $\varphi(T) = L$. Likewise, the *reducing essential $n \times n$ spectrum* of T is the set $R_e^n(T)$ consisting of all those matrices L in $\mathbf{M}_n$ for which there exists a non-degenerate (but not necessarily irreducible) n-dimensional representation ψ of $\mathcal{C}_e^*(T)$ such that $\psi(\pi(T)) = L$.

Our main objective in this paper is to obtain some further information concerning the sets $R^n(T)$ and $R_e^n(T)$ for a given operator T in $\mathcal{L}(\mathcal{H})$. In particular, we prove (Theorem 2.3) that these sets are upper semi-continuous functions of T with respect to the norm topology of $\mathcal{L}(\mathcal{H})$. In this study, special attention will be given to the class of n-normal operators in $\mathcal{L}(\mathcal{H})$. By definition, an n-normal operator in $\mathcal{L}(\mathcal{H})$ is an operator which is unitarily equivalent to an $n \times n$ operator matrix (acting in the usual fashion on the direct sum of n copies of $\mathcal{H}$) whose entries are commuting normal operators. We prove the following extension of Berg's theorem [3]: If T and S are n-normal operators in $\mathcal{L}(\mathcal{H})$, then T is unitarily equivalent to a compact perturbation of S if and only if

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