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# AN INTEGRAL FORMULA FOR HOLOMORPHIC FUNCTIONS ON STRICTLY PSEUDOCONVEX HYPERSURFACES

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**Introduction.** It is known from the work of Henkin [4] and of Ramírez de Arellano [8] that there is a useful integral formula for functions holomorphic on strictly pseudoconvex domains in $\mathbf{C}^n$. Granted the importance of this formula, it is natural to seek an analogous formula for domains in complex manifolds. The principal result of the present paper, Theorem II.1, is that such a formula exists for certain hypersurfaces in strongly pseudoconvex domains in $\mathbf{C}^n$. The formula we obtain, it may be noted, is valid not only for nonsingular hypersurfaces, but also for certain subvarieties which possess sufficiently restricted singular points.

Section II of the paper contains a careful statement of our integral formula together with its proof. Section I is devoted to the theory of Cauchy–Fantappiè forms; it is partially a recollection of known results.

**I. Cauchy-Fantappiè Forms.** If $\mathfrak{M}$ is an $N$-dimensional complex manifold and if $\phi = (\phi_1, \cdots, \phi_N) : \mathfrak{M} \to \mathbf{C}^N$ is a $\mathcal{C}^\infty$ map, we define forms $\omega(\phi)$ and $\omega'(\phi)$ by

$$\omega(\phi) = d\phi_1 \wedge \cdots \wedge d\phi_N$$

and

$$\omega'(\phi) = \sum_{i=1}^{\infty} (-1)^{i-1}\phi_i \, \bar\partial\phi_1 \wedge \cdots \wedge \widehat{\bar\partial\phi_i} \wedge \cdots \wedge \bar\partial\phi_N .$$

Then $\bar\partial\omega'(\phi) = N\bar\partial\phi_1 \wedge \cdots \wedge \bar\partial\phi_N$. If $f : \mathfrak{M} \to \mathbf{C}^N$ is a holomorphic map, the Cauchy–Fantappiè form $\Omega_{\phi;f}$ is defined by

$$\Omega_{\phi;f} = \frac{\omega'(\phi) \wedge \omega(f)}{(\phi_1 f_1 + \cdots + \phi_N f_N)^N} .$$

In the sequel, we shall frequently write $\phi \cdot f$ for $\phi_1 f_1 + \cdots + \phi_N f_N$ so that $\Omega_{\phi;f} = (\phi \cdot f)^{-N}\omega'(\phi) \wedge \omega(f)$.

The importance of Cauchy–Fantappiè forms stems from two facts: A) *The form $\Omega_{\phi;f}$ is closed.* B) *Given a second smooth map $\psi : \mathfrak{M} \to \mathbf{C}^N$, the forms $\Omega_{\phi;f}$ and $\Omega_{\psi;f}$ differ on their common domain of definition by an exact form.*

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