

EXPONENTIAL SUMS OVER FINITE SIMPLE JORDAN ALGEBRAS AND FINITE SIMPLE ASSOCIATIVE ALGEBRAS

LIANG-CHI TSAO

In [2], we obtained explicit formulas for certain exponential sums over finite simple Jordan algebras, and we applied them to the proof of the rationality of the Fourier coefficients of certain Eisenstein series on tube domains. The purpose of the present paper is to discuss several generalizations of those exponential sums. In particular, we derive formulas for certain exponential sums over finite simple associative algebras from the formulas for sums over finite simple Jordan algebras; the result itself is interesting, and, quite possibly, it will also be useful in other number-theoretic problems, for example, in the discussion of the Fourier coefficients of certain Eisenstein series on general linear groups.

In Section 1, we review the basic notions and results of exponential sums over finite simple Jordan algebras [2]. For details, the readers should consult [3, §10]. In Section 2, we discuss a possible direction of generalization in which matrix entries are replaced by elements from a commutative ring with a certain involution. In Section 3, we define certain characters of Jordan algebras, and in Section 4 we calculate the exponential sums attached to them. In Section 5, we obtain reduction formulas for certain exponential sums in the case of characteristic 2. Finally, in Section 6, we obtain explicit formulas of certain exponential sums over finite simple associative algebras. The basic observation is that we may have a mapping from a finite simple associative algebra to a corresponding simple Jordan algebra, and that this mapping relates the exponential sums over each of them and gives the desired results.

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1.1 Let F_q denote the finite field of q elements, where $q = p^\alpha$ and p is an odd prime. The finite central simple Jordan algebras J_q over F_q are

- (A) the algebras of $n \times n$ Hermitean symmetric matrices over a quadratic algebra over F_q ,
- (B) the algebras of $n \times n$ symmetric matrices over F_q ,
- (C) the algebras of $n \times n$ quaternion symmetric matrices over a quaternion algebra over F_q ,

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