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ON EXTREME POINTS AND SUPPORT POINTS FOR SOME FAMILIES OF UNIVALENT FUNCTIONS

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§1. Introduction. Let $\mathfrak{F}$ denote a compact set in the linear space of functions analytic in a plane domain D equipped with the topology of locally uniform convergence. A function f is an *extreme point* of $\mathfrak{F}$ if $f \in \mathfrak{F}$ and if

$$f = tf_1 + (1 - t)f_2$$

with $0 < t < 1$ and $f_1, f_2 \in \mathfrak{F}$ implies $f = f_1 = f_2$. Correspondingly, f is a *support point* of $\mathfrak{F}$ if $f \in \mathfrak{F}$ and there exists a continuous linear functional λ, non-constant on $\mathfrak{F}$, such that

$$\text{Re } \lambda(f) = \max_{\mathfrak{F}} \text{Re } \lambda.$$

We denote by ext $\mathfrak{F}$ and spt $\mathfrak{F}$ the set of extreme points and the set of support points of $\mathfrak{F}$, respectively.

From a geometric point of view ext $\mathfrak{F}$ and spt $\mathfrak{F}$ provide a great deal of information about $\mathfrak{F}$. Indeed, by the Krein–Milman Theorem the extreme points determine the closed convex hull of $\mathfrak{F}$ (which we shall denote by $\overline{co}\mathfrak{F}$), and through each support point passes a supporting hyperplane of $\mathfrak{F}$.

From the point of view of solving linear extremal problems, it is of course spt $\mathfrak{F}$ that one would like to know. However, it is often the case that ext $\mathfrak{F}$ is easier to determine, and by the Krein–Milman Theorem every linear extremal problem has a solution in ext $\mathfrak{F}$.

In recent years considerable work has been done on identifying the extreme points and support points for various families of schlicht functions (see [2, 3, 4, 5, 6, 10, 11, 16] for a representative sampling). In some instances complete characterizations of the set of extreme points have been found and this knowledge has led to elegant solutions to linear extremal problems [13, 14]. However, this approach is only successful when the set of extreme points is a relatively small set; when this is not the case one must deal directly with the problem of characterizing the support points.

In this note we shall be concerned with several classes of functions. Let Σ denote the class of functions $f(z) = (1/z) + \sum_{n=0}^{\infty} a_n z^n$ that are analytic and univalent in $0 < |z| < 1$ and Σ_0 the subclass with $a_0 = 0$. The *meromorphic starlike class* Σ^* consists of those functions in Σ that map $|z| < 1$ onto a domain whose complement is starlike with respect to the origin, and for $\alpha < 1$

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