Vol. 42, No. 2 DUKE MATHEMATICAL JOURNAL© June 1975

STRUCTURAL INSTABILITY AND EXTENSIONS OF RIEMANN SURFACES

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0. Introduction.

1. First we recall some notions and definitions. Throughout this paper a letter Q, R, S, T or Z will denote a Riemann surface defined by a pair $\langle M, \Phi \rangle$, where M is a two-dimensional connected manifold and Φ a maximal conformal atlas of M. Such an atlas Φ consists of conformally compatible charts φ, where each φ is a homeomorphism of some subdomain U of M into the complex plane E. Two Riemann surfaces R and S are equal, if and only if they are conformally equivalent, i.e., there is a conformal bijection of R onto S. Otherwise R and S are said to be distinct.

Suppose that f, $f : Q \to R$, is a conformal injection. Since Q is equal to $f(Q)$, one can identify Q with $f(Q)$ and hence one gets $Q \subset R$. We then say that f embeds Q into R, that Q is embedded in R, and that R is an extension of Q.

A Riemann surface is said to be maximal and denoted by Z, if and only if for every extension T of Z, the embedding f, $f : Z \to T$, is bijective. Obviously any compact R is maximal.

2. Almost fifty years ago S. Bochner [B] proved that to every R there exists a maximal extension Z. In general such a Z is not unique. So the problem arises to determine the class of Riemann surfaces that have a unique maximal extension. It is here understood that R belongs to that class if and only if any two maximal extensions Z_1 and Z_2 of R are equal.

In this paper that problem is solved and the corresponding class is characterized (Theorem 2). For the history of the problem as well as for references to existing partial solutions we refer to Remark 3.

3. As our principal tool we employ a technique of changing the structure of a Riemann surface with respect to certain small sets only. Such a technique can be developed independently and forms the main topic of this paper.

Throughout this paper the letter N will denote a non-empty, closed and totally disconnected subset of some Riemann surface. If N is in R, then the restriction Φ_N of the atlas Φ to the open, necessarily connected subset $M - N$ consists of all charts φ, $\varphi : U \to E$, satisfying the condition $U \subset M - N$. Now we want to extend the atlas Φ_N of $M - N$ to a maximal conformal atlas Ψ of M. Two main problems appear in this context. When is Φ the only possible ex-

Received July 26, 1974. Revision received October 28, 1974.