ALMOST ISOMETRIES OF BANACH SPACES AND MODULI OF RIEMANN SURFACES II

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1. Introduction and summary. Let $\mathcal{S}$ denote the set of compact bordered Riemann surfaces. For $\bar{S}$ in $\mathcal{S}$ denote the border of $\bar{S}$ by $\partial\bar{S}$ and the interior of $\bar{S}$ by S. For $\bar{S}$ in $\mathcal{S}$, $R(\bar{S})$, the Riemann space of $\bar{S}$, is the space of conformal equivalence classes of elements of $\mathcal{S}$ homeomorphic to $\bar{S}$. We will generally denote an element of $R(\bar{S})$ by a member of the equivalence class. The moduli topology on $R(\bar{S})$ is the topology induced by the Teichmüller metric, $D(\bar{S}_1, \bar{S}_2) = \inf\{\log K$: there is a quasiconformal homeomorphism of S_1 onto S_2 with maximal dilatation $K\}$. (For a discussion of quasiconformal maps see Ahlfors [1]. For a discussion of the Teichmüller metric in this context see Earle [4].)

For $\bar{S}$ in $\mathcal{S}$ let $A(S)$ be the supremum normed Banach algebra of functions continuous on $\bar{S}$ and analytic on S. For $\bar{S}_1$ and $\bar{S}_2$ in $\mathcal{S}$ let $L(A(S_1), A(S_2))$ be the set of continuous invertible linear maps of $A(S_1)$ onto $A(S_2)$. For T in $L(A(S_1), A(S_2))$ let $c(T) = \|T\|\,\|T^{-1}\|$. Note that $c(T)$ can reasonably be thought of as the dilatation of the quasiconformal homeomorphism T. Define a distance between elements of $\mathcal{S}$ by $d(\bar{S}_1, \bar{S}_2) = \inf\{\log c(T)\colon T \in L(A(S_1), A(S_2))\}$.

In a previous paper [7] the author proved the following theorem.

THEOREM 1. *Let $\bar{S}$ be a planar element of $\mathcal{S}$. d is a metric on $R(\bar{S})$. The topology induced by this metric is the moduli topology.*

In this paper Theorem 1 is extended to non-planar surfaces. The following is proved.

THEOREM 2. *Let $\bar{S}$ be an element of $\mathcal{S}$. d is a metric on $R(\bar{S})$. The topology induced by this metric is the moduli topology.*

The previous paper with this title [8] contains some of the component results needed to establish Theorem 2. In particular it is shown that d is a metric and the topology induced by d is coarser than the moduli topology. Thus, to complete the proof of Theorem 2 it suffices to prove the following.

THEOREM 3. *Let $\bar{S}$ be an element of $\mathcal{S}$ and $\bar{S}_1, \bar{S}_2, \cdots$ elements of $\mathcal{S}$ homeomorphic to $\bar{S}$. If $\lim_{n\to\infty} d(\bar{S}_n, \bar{S}) = 0$ then $\lim_{n\to\infty} D(\bar{S}_n, \bar{S}) = 0$.*

Sections 2, 3, and 4 contain preliminary results. The proof of Theorem 3 is in Section 5. Section 6 contains notes and comments.

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