GLOBAL SUPPORT PROPERTIES OF STATIONARY ERGODIC PROCESSES

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Nelson [7] has made the deep observation that a variety of quantum fields analytically continued to imaginary time are represented by stationary, ergodic, generalized Markov processes (see also [12, 14]). Recently, there has been some interest in determining the support properties of the measure in the canonical model of the process on $C_0^\infty(\mathbf{R}^n)'$ given by Minlos' theorem [4]. The analysis has begun with the free Euclidean field, i.e., the Gaussian process over $C_0^\infty(\mathbf{R}^n)$ with covariance

$$\int q(f)q(g)\, d\mu_0 = \langle f, (-\Delta + m_0^2)^{-1} g\rangle .$$

The main results found for μ_0 are:

(0) ([2]) Those distributions equal to a signed measure on some open set have measure 0 if $n \geq 2$.

(1) ([1, 2]) If $\tilde{\Delta}$ is the Laplacian in $n - 1$ dimensions, then μ_0 is supported on $(-\tilde{\Delta} + 1)^\alpha H$ if $\alpha > n/4 - 1/2$ and H is the set of locally Hölder continuous functions.

(2) ([2]) If $f \in C_0^\infty(\mathbf{R}^n)$ and $q_f(x) = q(f(\cdot - x))$, then with μ_0-probability one:

$$\overline{\lim_{x\to\infty}}\; q_f(x)/\sqrt{\ln |x|} = C(f, m_0, n)$$

where C is an explicit constant only depending on f, m_0, n.

(3) ([9]) μ_0 is supported by $(-\tilde{\Delta} + 1)^\alpha (1 + x^2)^{n/4} [\log (2 + x^2)]^{\beta/2} L^2(\mathbf{R}^n)$ if $\alpha > \frac{1}{4}n - \frac{1}{2}$ and $\beta > 1$ and by its complement if $\alpha > \frac{1}{2}n - 1$ and $\beta < 1$.

We have studied the extension of these results to the $P(\phi)_1$ and $P(\phi)_2$ Markov fields [7, 6, 14]. (0), (1) extend (or should extend) to these theories since they are known or believed to be locally absolutely continuous to the free theory. (This local absolute continuity is known for all $P(\phi)_1$ theories [6] and for "small coupling" $P(\phi)_2$ [8]). We will examine the analog of (2) in detail elsewhere [11] using partly methods from [10]. This note had its genesis in an attempt to extend the result (3) of M. Reed and L. Rosen to these interacting fields. We have found that their result only depends on the ergodicity of the process associated with μ_0.

Our results all follow from Theorem 1 which is a simple consequence of the ergodic theorem:

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