

# ON CARTAN'S METHOD OF LIE GROUPS AND MOVING FRAMES AS APPLIED TO UNIQUENESS AND EXISTENCE QUESTIONS IN DIFFERENTIAL GEOMETRY

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## 0. INTRODUCTION.

These notes are an exposition of the philosophy due to Elie Cartan that, via the use of moving frames, the theory of Lie groups constitutes a powerful and elegant method for studying uniqueness and existence questions for submanifolds of a homogeneous space. This philosophy, as expounded in his beautiful book "Groupes finis et continus et la géométrie différentielle", Gauthier-Villars (Paris), is perhaps not as widely appreciated as it should be, especially as regards the higher order invariants of a submanifold. A possible reason for this is that, even though the basic Lie group statements underlying the theory are of a rather general nature, their application to geometry seems at present more adapted to special cases depending on subtle conditions of non-degeneracy, rather than constituting a vast general theory. It is the intricacy and beauty of these special cases which in the end justifies the general approach. Our purpose here is to present a somewhat updated and hopefully clear exposition of a portion of the Cartan philosophy, together with a few traditional and some new applications to geometry. In particular, we emphasize the case of holo-

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