ERGODIC AUTOMORPHISMS AND LINEAR SPACES OF OPERATORS

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Introduction. This paper investigates some of the relations between ergodic theory and operator theory. To each ergodic measure preserving automorphism α on a Lebesgue measure space X one associates a unitary operator U_α acting on $L^2(X, m)$. As is well known, the unitary equivalence class of U_α is not enough to determine the conjugacy class of α. However, if we consider a larger family of operators, then the conjugacy class of α can be determined. Let $\mathfrak{M}$ be the maximal self-adjoint algebra of all L^∞-multiplication operators on $L^2(X, m)$. Arveson, in [2], shows that the unitary equivalence class of the Banach algebra, $\mathfrak{J}_\alpha$, generated by $\mathfrak{M}$ and U_α determines the conjugacy class of α, and vice versa. Further, it is shown in [6] that if two such Banach algebras $\mathfrak{J}_\alpha$ and $\mathfrak{J}_\beta$ are isometric, then either α is conjugate to β or α is conjugate to β^{-1}.

It is the purpose of this paper to prove similar results using a smaller family of operators, namely the linear manifold $\mathfrak{N}_\alpha$ generated by $\mathfrak{M}$ and U_α. Of course, by the result quoted above, the unitary equivalence of $\mathfrak{N}_\alpha$ and $\mathfrak{N}_\beta$ implies the conjugacy of α and β. We replace the unitary equivalence of $\mathfrak{N}_\alpha$ and $\mathfrak{N}_\beta$ by the a priori weaker condition that they be completely isometric (see section 2 for a definition) and show that this implies α and β are conjugate. This result is readily obtained using results from [1]. The main theorem in this paper states that if $\mathfrak{N}_\alpha$ and $\mathfrak{N}_\beta$ are isometric then either α is conjugate to β or α is conjugate to β^{-1}. Since there is no evident way to extend an isometry on $\mathfrak{N}_\alpha$ to one on $\mathfrak{J}_\alpha$, the results of [6] cannot be utilized. Instead, we must investigate the nature of such an isometry and its relationship with the automorphisms α and β. This in turn requires a study of the interaction of U_α and certain multiplication operators, in particular, the projections in $\mathfrak{M}$.

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1. Notation and definitions. Throughout this paper (X, m) will denote a fixed Lebesgue measure space with $m(X) = 1$. All subsets of X which appear will be measurable, either by assumption or by a standard argument, whichever is appropriate. We omit the arguments and we invariably omit the adjective "measurable." A measure preserving automorphism α on X is said to be *ergodic* if each of its invariant subsets is either a null set or differs from X by a null set. Two ergodic measure preserving automorphisms α and β are *conjugate* if there exists a (necessarily measure preserving) automorphism σ such that $\alpha = \sigma\beta\sigma^{-1}$.

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