BESSEL FUNCTIONS AS p-ADIC FUNCTIONS OF THE ARGUMENT

B. DWORK

Dedicated (prematurely) to Philip Hartman on his 60th birthday.

1. Introduction. It has been known for some time [3, Theorem 3.4] that if the coefficients of a power series $F = \sum_{m=0}^{\infty} A(m)t^m \in \mathbf{Z}_p[[t]]$ satisfy certain congruence conditions:

$$A(0) = 1$$

$$(1.1) \qquad A(c + p\mu)/A(\mu) \in \mathbf{Z}_p$$

$$\frac{A(c + p\mu + mp^{s+1})}{A(\mu + mp^s)} \equiv \frac{A(c + p\mu)}{A(\mu)} \bmod p^{s+1}$$

for all $c \in [0, p)$, $\mu \in \mathbf{Z}_+$, $m \in \mathbf{Z}_+$ then

$$(1.2) \qquad f(x) = F(x)/F(x^p)$$

$$(1.3) \qquad \eta = F'/\psi$$

may be prolonged analytically to the set

$$\mathfrak{D} = \left\{x \mid \left|\sum_{m=0}^{p-1} A(m)x^m\right| = 1\right\}$$

Furthermore for $a \in \mathfrak{D}$, the local solution u_a of the differential equation

$$(1.4) \qquad u_a' = \eta u_a$$

$$u_a(a) = 1$$

converges and is bounded in the disk $D(a, 1^-)$ $(= \{x | |x - a| < 1\})$, and equation (1.2) generalizes to the form

$$f(x)/f(a) = u_a(x)/u_{a^p}(x^p)$$

for all $x \in D(a, 1^-)$.

This theory was first applied [3] to the hypergeometric function $F(\frac{1}{2}, \frac{1}{2}, 1; x)$ $(p \neq 2)$ but it may be equally well applied to the Bessel function J_0

$$J_0(x) = \sum_{m=0}^{\infty} (-1)^m \left(\left(\frac{x}{2}\right)^m \frac{1}{m!}\right)^2$$

provided one changes the variable so as to obtain a function which converges

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