# ABSOLUTE CONVERGENCE OF FOURIER SERIES ON CERTAIN GROUPS, II

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In this paper we present a number of new results on the absolute convergence of the Fourier series of functions defined on a so-called Vilenkin group, that is, a compact, metrizable, 0-dimensional, abelian group $G$. Examples of such groups are (i) the dyadic group $2^\omega$, which has the set of Walsh functions as its character group and (ii) the additive group of the ring of integers in any $p$-adic or $p$-series field. In [2] we studied functions on $G$ which satisfied a Lipschitz condition of order $\alpha$ with $0 < \alpha \leq 1$. In the present paper we are concerned with functions belonging to a class Lip $(\alpha, p)$ with $0 < \alpha \leq 1$ and $1 \leq p$ (see Definition 2). For $\beta > 0$ we shall denote the set of all functions $f$ in $L_1(G)$ for which $\sum_{n=1}^\infty |\hat{f}(n)|^\beta < \infty$ by $A(\beta)$.

The main result of Section 1 is Theorem 1 in which we give a sufficient condition, expressed in terms of the integrated modulus of continuity, for a function $f$ in $L_p(G)$, $1 \leq p \leq 2$, to belong to $A(\beta)$ for certain values of $\beta$. As a corollary to this theorem we obtain the analogue on $G$ of a well-known result of Szász for trigonometric Fourier series, namely, if $f \in$ Lip $(\alpha, p)$ for some $\alpha$ and $p$ with $0 < \alpha \leq 1$ and $1 \leq p \leq 2$, then $f \in A(\beta)$ for all $\beta > p/(\alpha p + p - 1)$. In Theorem 2 we prove that Szász's result cannot be extended to

$$\beta = p/(\alpha p + p - 1).$$

In Section 2 we investigate the problem of how to ameliorate functions in Lip $(\alpha, p)$, which do not necessarily have an absolutely convergent Fourier series, into functions belonging to $A(1)$ by means of the convolution operator. In Theorems 4 and 5 we consider convolutions $f * g$ with $f \in$ Lip $(\alpha, p)$ and $g \in$ Lip $(\beta, q)$ for certain pairs $(\alpha, p)$ and $(\beta, q)$, whereas in Theorem 6 we consider the convolution of functions in Lip $(\alpha, p)$ with functions in $L_q(G)$.

The results of this paper, which are all formulated in terms of functions defined on a Vilenkin group, are also true for functions defined on the unit circle and their (trigonometric) Fourier series. The proofs in this case are very similar to the proofs given here, and, hence, we shall not provide further details for this case.

**1.** Except for some minor modifications we shall use the same notation and terminology as in [2]. For any number $p$ such that $1 \leq p < \infty$, we denote its conjugate by $p'$, similarly for $q$ and $q'$, etc. We shall use the symbol $C$ to denote a constant, which may assume different values in different formulas. Further-

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