M-HYPONORMAL OPERATORS

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A bounded linear operator T on a Hilbert space H is called an *M-hyponormal operator* if there exists a real number M such that $||(T - zI)^*x|| \leq M\ ||(T - zI)x||$ for all $x \in H$ and for all $z \in \mathbf{C}$ (the field of complex numbers). The notion of an M-hyponormal operator is due to Stampfli (unpublished) and is a generalization of the notion of a hyponormal operator. Unlike a hyponormal operator, an M-hyponormal operator T may have the spectral radius strictly less than $||T||$. However, we shall show that an M-hyponormal operator T, such that $\sigma(T) = \sigma_c(T)$ is contained in a rectifiable Jordan curve satisfies a strong version of Dunford's boundedness condition (B) [2, page 2138], where $\sigma(T)$ and $\sigma_c(T)$ denote the spectrum and continuous spectrum of T respectively. This generalizes a result of Stampfli [6] about hyponormal operators with spectrum contained in a C^2 Jordan curve. Finally, using the results of the author [7], we shall show that an M-hyponormal operator T with $\sigma(T) = \sigma_c(T)$ contained in C^1 Jordan curve and satisfying the growth condition (G_1) is normal.

The proofs of the following results follow either from the definition of an M-hyponormal operator or are similar to the proofs of the corresponding results about hyponormal operators. Note that if T is an M-hyponormal operator then $M \geq 1$ and T is hyponormal iff $M = 1$.

PROPOSITION 1. *T is an M-hyponormal operator iff*

$$M^2(T - z)^*(T - z) - (T - z)(T - z)^* \geq 0 \quad \text{for all} \quad z \in \mathbf{C}.$$

PROPOSITION 2. *If T is an M-hyponormal operator, then*

(i) *$Tx = zx$ implies that $T^*x = \bar{z}x$;*

(ii) $||(T^* - \bar{z})^{-1}x|| \leq M\ ||(T - z)^{-1}x||$ for all z in resolvent set of T; and

(iii) $||(T - z)x||^{n+1} \leq M^{n(n+1)/2}\ ||(T - z)^{n+1}x||$

PROPOSITION 3. *Let T be an M-hyponormal operator.*

(i) *If $(T - z)^nx = 0$ then $(T - z)x = 0$.*

(ii) *If $Tx = z_1x$ and $Ty = z_2y$, $z_1 \neq z_2$ then $(x, y) = 0$.*

(iii) *If there exists a polynomial $p(z)$ such that $p(T) = 0$ then T is normal.*

(iv) *If H is finite dimensional then T is normal.*

We shall now give an example of an M-hyponormal operator which is not hyponormal.

Example. Let $\{e_i\}_{i=1}^{\infty}$ be an orthogonal basis of a Hilbert space H. Let T be a weighted shift defined by $Te_1 = e_2$, $Te_2 = 2e_3$ and $Te_i = e_{i+1}$ for $i \geq 3$.

Received July 9, 1973. Revised version received February 22, 1974. This work constitutes a part of the author's Ph.D. thesis written at Indiana University under the direction of Professor Joseph G. Stampfli and was supported by the National Science Foundation.