REMARKS ON THE FOUR AND FIVE DIMENSIONAL s-COBORDISM CONJECTURES

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1. Introduction. In [4] Siebenmann proves that either the 4-dimensional or the 5-dimensional s-cobordism conjectures, possibly both, are false in the PL and DIFF categories. His result is

THEOREM 0 ([4]). *Fix $k = 0, 1, 2$ and choose the* DIFF *or* PL *category. Then there exists an h-cobordism (W, V, V') of the following description:*

a) *V is $D^k \times T^{4-k}$ or $D^k \times T^{3-k}$ (perhaps both possible).*
b) *$V' \simeq V$.*
c) *(W, V, V') is an invertible cobordism.*
d) *There exists a topological homomorphism $e: V \times (I; 0, 1) \to (W; V, V')$ that gives an isomorphism $\partial(V \times I) \to \partial W$.*
e) *(W, V, V') is not a product cobordism. In fact, no finite odd covering of it is a product cobordism (and no finite even covering either if $k = 2$).*

In the statement D^k denotes the k-dimensional disk and T^n denotes the product of n circles. Siebenmann's approach is to use the results of Hsiang and Shaneson [2] and to derive a contradiction to them if every invertible cobordism between $D^k \times T^{3-k}$ and itself and between $D^k \times T^{4-k}$ and itself were a product.

In this note we wish to examine this result and to provide a different perspective towards it. In particular, we wish to show that in a precise sense there is a failure in dimension 4 or in dimension 5, but not in both dimensions. Our starting point will be the results of [3]. We will need them for the case of a manifold with boundary where everything is trivial along the boundary; it is straightforward to adapt the proofs in [3] to this case. We also start with the results of [2], although in an apparently weaker form than used in [4].

We first establish some notation (cf. [3], [5]). We will work in the PL category; similar results can be obtained in the DIFF category. Let W denote a compact manifold (with or without boundary). By $\mathcal{A}(W)$ let us denote the pseudoisotopy classes of PL homeomorphisms of W which are the identity on ∂W; here a pseudoisotopy must restrict to the identity pseudoisotopy on the boundary. $\mathcal{A}(W)$ forms a group under composition. Let $\mathcal{A}_0(W)$ denote the subgroup of $\mathcal{A}(W)$ generated by PL homeomorphisms homotopic rel ∂W to the identity. Let $IC(W)$ denote the equivalence classes of invertible cobordisms from W to itself (henceforth called $W - IC$) which are trivial on the boundary; i.e., (U, W, W, j_0, j_1, J), where $A = \partial U \backslash \text{int}(U_0 \cup U_1)$ and $j_n: W \to U_n$, $J: \partial W \times I \to A$ are PL homeomorphisms such that $J(x, n) = j_n(x)$, $n = 0, 1$. $(U, W, W,$

Received September 4, 1973. Revised version received May 9, 1974. This research was supported in part by a grant from the Tulane University Committee on Research.