THE PFLUGER BOUNDARY OF A BOUNDED PLANE DOMAIN

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Introduction. Since Carathéodory's original prime end construction [3] a number of additional boundary constructions have appeared, some which are restricted to simply connected domains [6], [12], [11], and some for more general domains [5], [7], including Riemann surfaces [4]. Recently, Arsove and Johnson [1], [2] developed a boundary by a very geometrical construction.

In [9] Pfluger suggested that prime ends be representable as maximal connected harmonic null sets. The author has successfully carried out this suggestion to determine the Pfluger compactification of an arbitrary bounded plane domain.

The purpose of this paper is to use the development of Arsove and Johnson to describe the Pfluger boundary and to give a condition when it agrees with the Arsove–Johnson boundary. Both of these boundaries reduce to Carathéodory's prime ends if the domain is simply connected.

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1. Preliminaries. Ω is a bounded domain in the plane with Euclidean boundary $\partial\Omega$. A component of $\partial\Omega$ containing more than one point is a *boundary continuum*; otherwise the boundary component is a *single boundary point*. In general, Ω is infinitely connected.

The unit disc is denoted by U and its boundary, the unit circle, by T. A *circular slit disc* Σ is a domain contained in U and satisfying

(i) $0 \in \Sigma$,

(ii) T is the outer boundary of Σ,

(iii) every inner boundary component of Σ is either a single point or a closed subarc σ of a circle centered at 0.

The circular slits and boundary points of Σ are denoted by S; hence $\partial\Omega = S \cup T$.

We distinguish three subsets of T. Denote by $A = T - \bar{S}$ the set of all *free* boundary points in T, by C the points in T which are inaccessible from Σ, and by B the complement of $A \cup C$ in T. Since A and C are both open in T, B and $B \cup C$ are closed in T.

THEOREM (Reich and Warschawski). *Suppose that D is a bounded plane domain containing the origin and define $\mathfrak{F}^*(D)$ to be the set of all functions f such that*

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