INTEGRAL FORMULAE FOR PLURIHARMONIC FUNCTIONS ON BOUNDED SYMMETRIC DOMAINS

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Let D be a bounded symmetric domain with the Bergman–Shilov boundary B and $0 \in D$. For $1 \leq p < \infty$, let $ph^p(D)$ denote the space of pluriharmonic functions h on D for which $\sup_{0<r<1} \int_B |h_r(t)|^p \, d\mu(t) < \infty$, and $ph^\infty(D)$ the space of bounded pluriharmonic functions on D. As usual, $H^p(D)$ denotes the corresponding spaces of holomorphic functions on D. In this note we prove a Poisson integral formula for functions in $ph^p(D)$ analogous to the results of [5, 10]. Furthermore, we also show the existence of a pluriharmonic kernel function on D and give integral formulae for functions in $ph^p(D)$ and $H^p(D)$ in terms of the pluriharmonic kernel and the boundary values of the functions on the Bergman–Shilov boundary. Some results concerning the pluriharmonic conjugates of functions in $ph^p(D)$ are also included.

1. Definitions and preliminary results. Let D be a bounded symmetric domain in $\mathbf{C}^n$ containing the origin, G the connected component of the identity of the group of holomorphic automorphisms of D and K the isotropy subgroup of G at the origin. Then G is a semi-simple Lie group, K a maximal compact subgroup and $D = G/K$ is a Hermitian symmetric space of noncompact type. The group G is transitive on D and extends continuously to ∂D, the topological boundary of D.

A bounded symmetric domain D is circular and starlike, that is, $tz \in D$ when $z \in D$ and $t \in \mathbf{C}$, $|t| \leq 1$ [11]. Let B denote the Bergman–Shilov boundary of D. B is circular and invariant under G. Furthermore K is transitive on B and consequently B has a unique normalized K-invariant measure μ. Throughout this note this measure will always be denoted by μ.

A continuous function h defined on D is pluriharmonic if for every holomorphic mapping γ of the unit disc U in $\mathbf{C}^1$ into D, $h \circ \gamma$ is harmonic in U [3, p. 271]. Since D is simply connected [7, p. 311], every pluriharmonic function h on D is the real part of a holomorphic function on D [13, p. 44].

We denote by $ph(D)$ and $H(D)$ the spaces of pluriharmonic and holomorphic functions on D respectively. For $1 \leq p \leq \infty$ we set

$$ph^p = ph^p(D) = \{h \in ph(D) : ||h||_p = \sup_{0<r<1} M_p(h, r) < \infty\}$$

where for $0 < r < 1$,

$$M_p(h, r) = \left\{\int_B |h(rt)|^p \, d\mu(t)\right\}^{1/p}, \qquad 1 \leq p < \infty,$$

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