BOUNDING A FREE ACTION OF AN ABELIAN GROUP

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1. Introduction. In this paper G will always denote a finite abelian group, and "G-action" means a smooth, orientation-preserving action of G on a compact, oriented manifold.

Let $\Omega_*(G)$ be the bordism of all such G-actions, and let $\hat{\Omega}_*(G)$ be the corresponding bordism of free G-actions. This notation, and full definitions, will be found in [4]. Consider the forgetful homomorphism

$$s : \hat{\Omega}_*(G) \to \Omega_*(G).$$

The image of s contains the element $\gamma(G) \in \Omega_0(G)$, which is represented by G, acting on itself by translation.

THEOREM 1. *Im s is generated by $\gamma(G)$ as in Ω_*-module, and is isomorphic to*
(a) Ω_* , *if G has odd order, and*
(b) $\Omega_*/torsion$, *if G has even order.*

THEOREM 2. *If $\varphi : G \times M \to M$ is any free G-action, then $[M, \varphi] = [M/G]\gamma(G) \in \Omega_*(G)$.*

Theorem 2 extends a well-known result of Conner and Floyd [2, (19.4)], who proved that $[M] = o(G)[M/G] \in \Omega_*$, where $o(G)$ is the order of G.

Theorem 1(a) is an easy consequence of Stong's theorem [4, Prop. 5.3] for the case of abelian p-groups, p an odd prime. Theorem 1(b) requires somewhat different methods. The only case known previously was $G = Z_2$, which is due to Rosenzweig [3]. Our proof of Proposition 6 includes his proof as a special case.

It is my pleasure to thank R. E. Stong for a useful conversation on these matters. As will be noted below, Stong has also proved a restricted version of Proposition 6.

2. Some preliminaries. If $H \subseteq G$ is a subgroup, then $e_G{}^H$, or simply e, will denote any of the extension homomorphisms of Conner and Floyd, as defined for example in [5, section 4]. Specifically, if (M, φ) is an H-action, then H acts on $G \times M$ by $h(g, m) = (gh^{-1}, hm)$ and G acts on $(G \times M)/H$ by $g[g', m] = [gg', m]$. The latter action is $e_G{}^H(M, \varphi)$. This construction is natural with respect to changes in isotropy condition; in particular, $se = es$.

There is a homomorphism $\hat{\Omega}_*(G) \to \Omega_*(BG)$ which assigns to $[M, \varphi]$ the class of a classifying map $M/G \to BG$. This is an isomorphism [2, (19.1)], so by its use one obtains homomorphisms

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