INTEGRATION BY PARTS FOR ABSTRACT WIENER MEASURES

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1. Introduction. Let C be the Banach space of real-valued continuous functions on [0, 1] vanishing at 0. Cameron [1] proved an integration by parts formula for the Wiener measure in C. Donsker [4], using different methods, proved this formula for Gaussian measures and applied it to study Fréchet–Volterra differential equations.

The purpose of this paper is to generalize this formula to abstract Wiener space [5]. A fundamental formula like this seems to be desirable for the calculus of infinitely many variables. Our proof, motivated by [6, Proposition 9], is much simpler and more transparent. We apply this formula to evaluate certain integrals. A more important application is to the Fourier-Wiener transform [2; 9]. In particular, we will be able to solve the following differential equation,

$$\partial_t u(t, x) = \text{trace } \partial_{xx} u(t, x) - 1/2\langle \partial_x u(t, x), x\rangle$$

with the initial condition $u(0, x) = f(x)$.

2. Integration by Parts. (H, B) will denote a fixed abstract Wiener space throughout this paper. $|\cdot|$ and $\|\cdot\|$ denote H-norm and B-norm, respectively. $\langle\ ,\ \rangle$ denotes the inner product of H and $(\ ,\ )$ the natural pairing between B^* and B. ($B^* \subset H \subset B$ as in [6]). $p_t(dx)$ denotes the Wiener measure in B with variance parameter t and $p_t(h, dx) = p_t(dx - h)$.

Besides the usual Fréchet differentiability, we will consider H-differentiability introduced in [6]. A function u from an open set U of B into a Banach space X is said to be H-differentiable at x if there exists a (unique) bounded linear operator, denoted by $Du(x)$, from H into X such that $\|u(x + h) - u(x) - Du(x)h\|_X = o(|h|)$, $h \in H$, where $\|\cdot\|_X$ is the norm of X. u is C_H^1 if $Du(x)$ exists for each x in B and Du is continuous from B into $L(H, X)$ (operator norm topology for $L(H, X)$). H-derivatives of higher order, denoted by $D^k u(x)$, and C_H^k can be defined inductively, $k \geq 2$. The usual Fréchet derivatives will be denoted by primes, such as $u'(x)$, $u''(x)$, $\cdots$ etc.

PROPOSITION 1. *Let f be a function from B into a Hilbert space K. Assume that* (i) $\int_B (\|f(x)\|_K)^2 p_t(dx) < \infty$ *and* (ii) *there exist constants $r > 0$ and $M < \infty$ such that $\int_B \|f(x)\|_K p_t(h, dx) < M$ for all $|h| < r$. Then the function g from the open set $V = \{h \in H\ ;\ |h| < r\}$ of H into K defined by*

$$g(h) = \int_B f(x) p_t(h, dx)$$

Received October 5, 1973. Research supported in part by NSF Grant GP-38010.